Fully Scalable MPC Algorithms for Embedded Planar Graphs

Graph drawing for planar graph algorithms

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UIUC Theory Seminar

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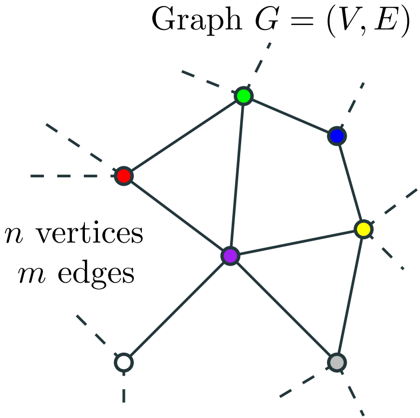
Massively parallel computing with graphs



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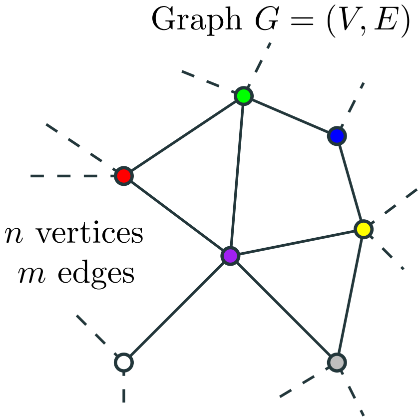

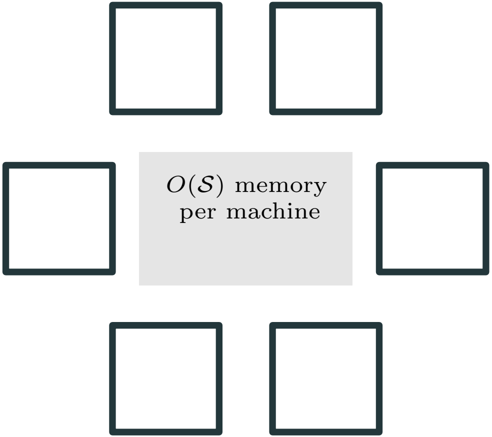

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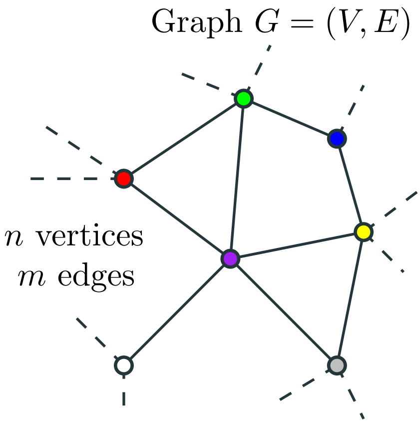

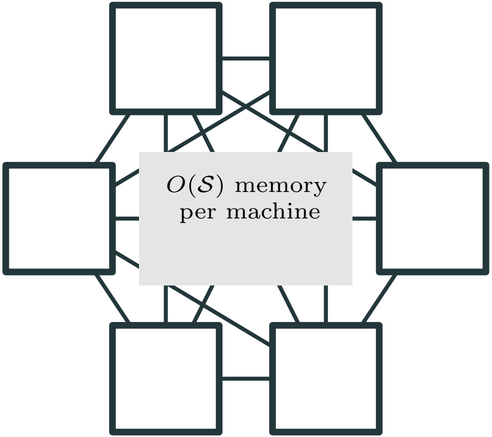

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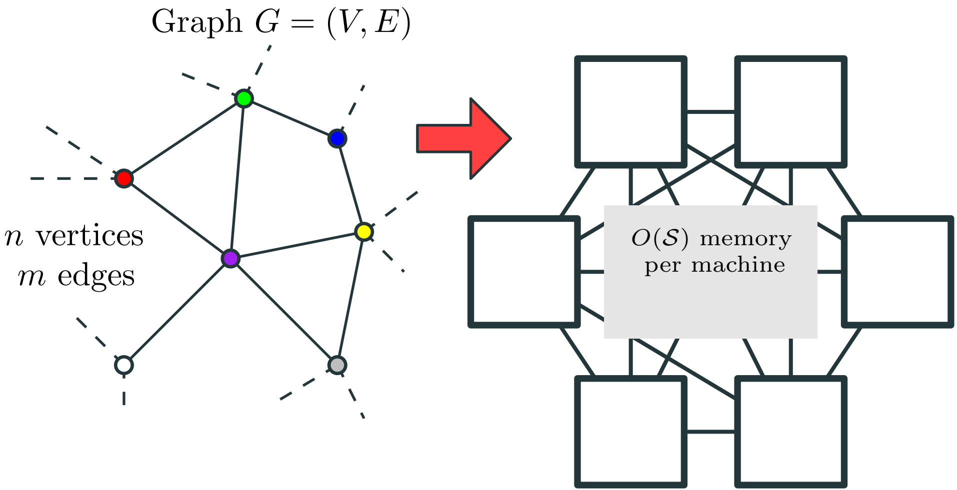

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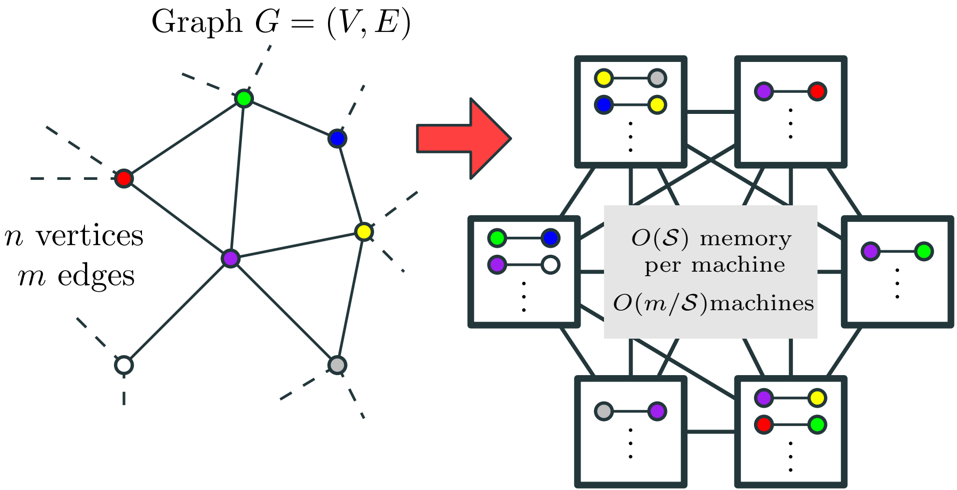

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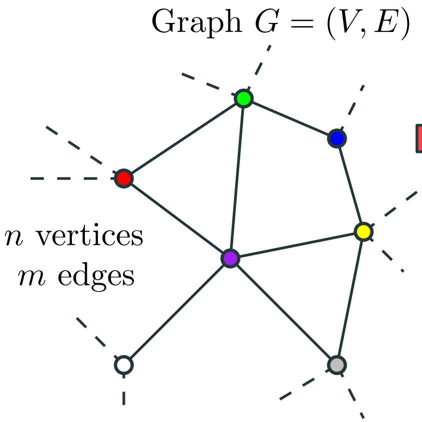

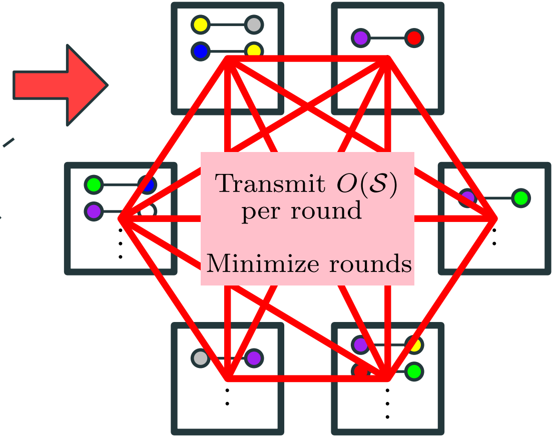

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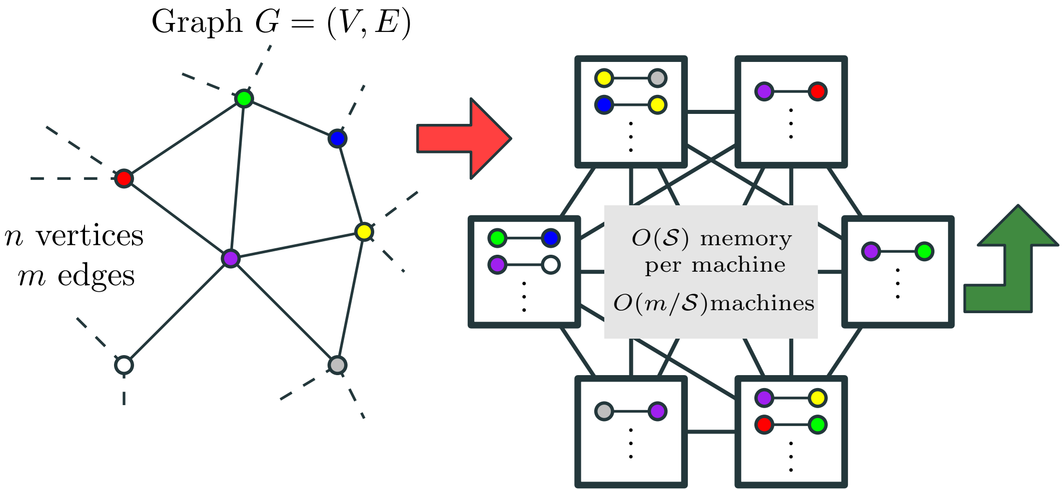

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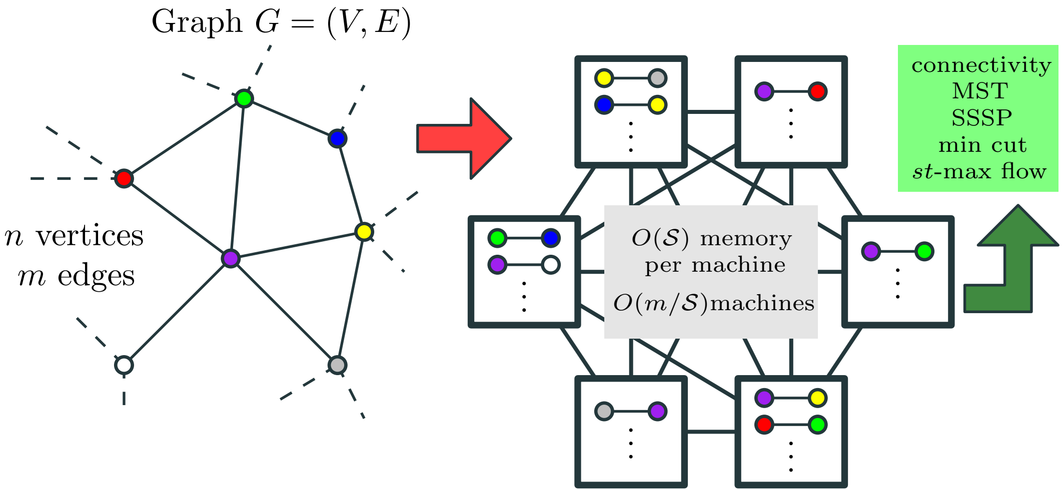
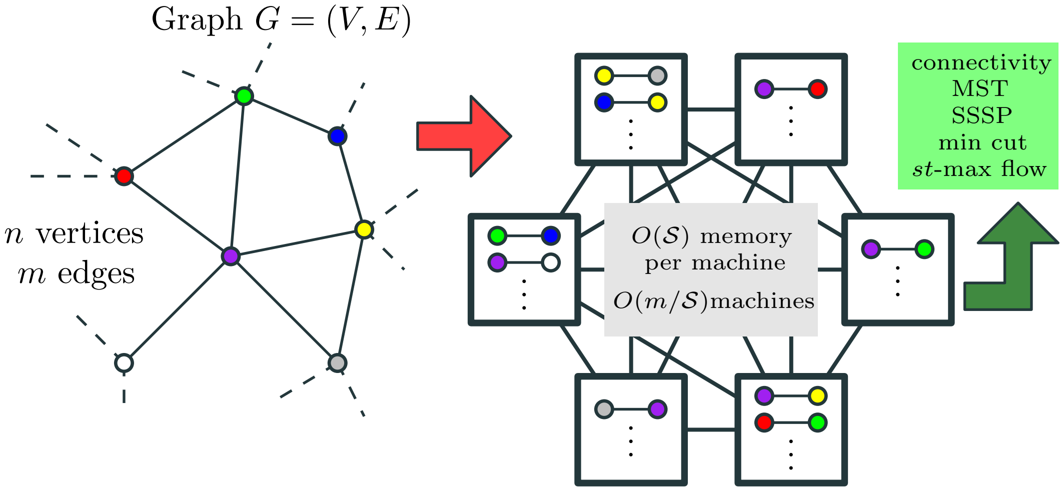
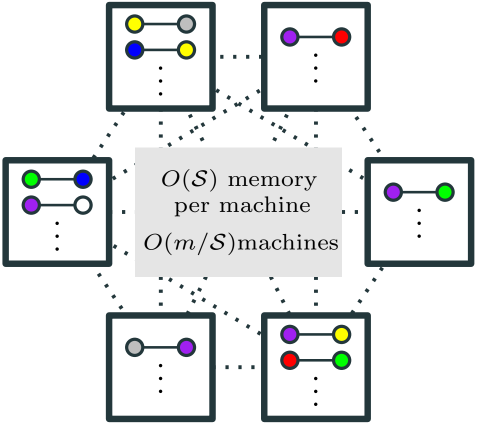
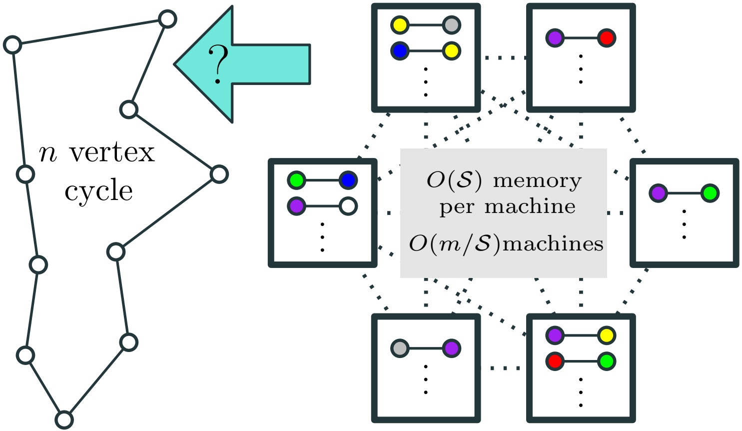


Definition
We say an algorithm is **fully scalable** if it works with $\mathcal{S} = n^{\delta}$ for any $\delta > 0$.

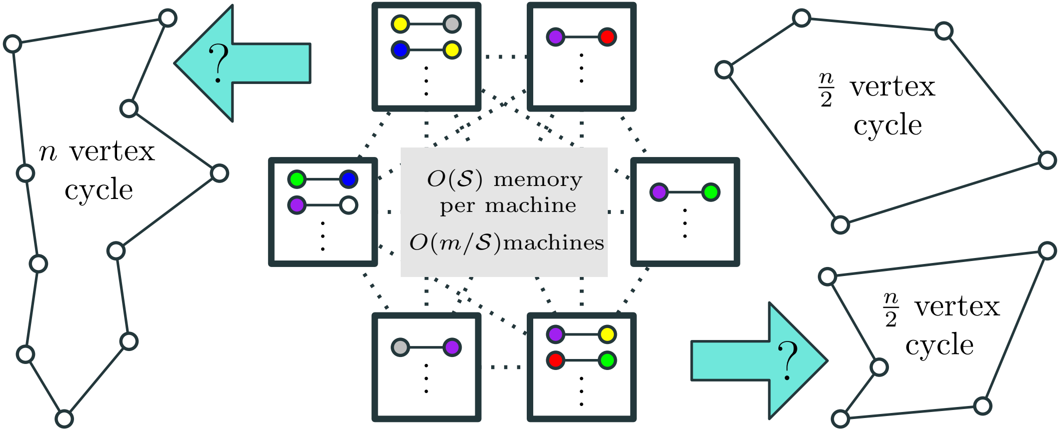


A bottleneck: The 1-vs-2 cycle conjecture



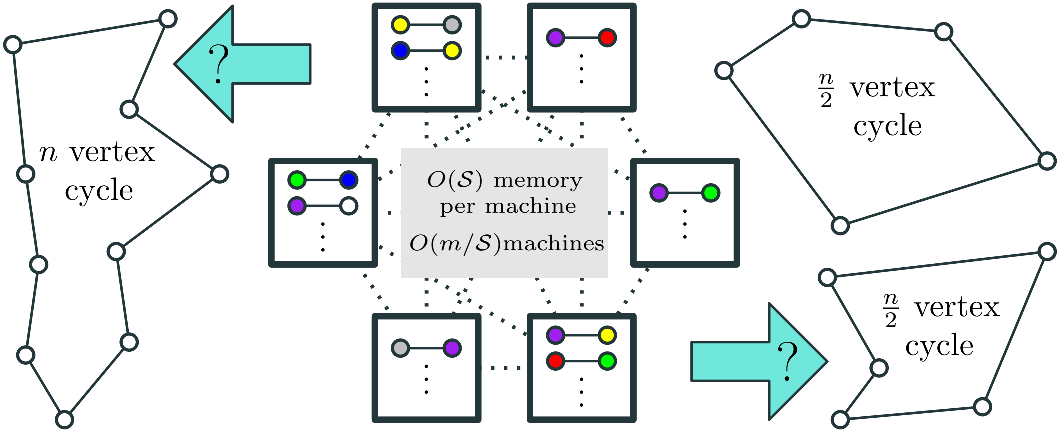

n vertex
cycle
?
$O(\mathcal{S})$ memory
per machine
$O(m/\mathcal{S})$machines

A bottleneck: The 1-vs-2 cycle conjecture



A bottleneck: The 1-vs-2 cycle conjecture

If $\mathcal{S} = O(n^{1-\varepsilon})$, then need $\Omega(\log n)$ rounds.



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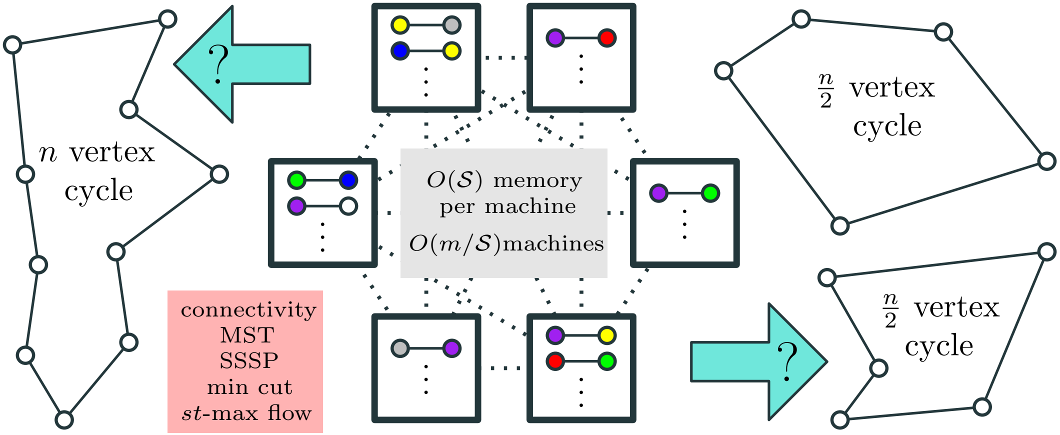


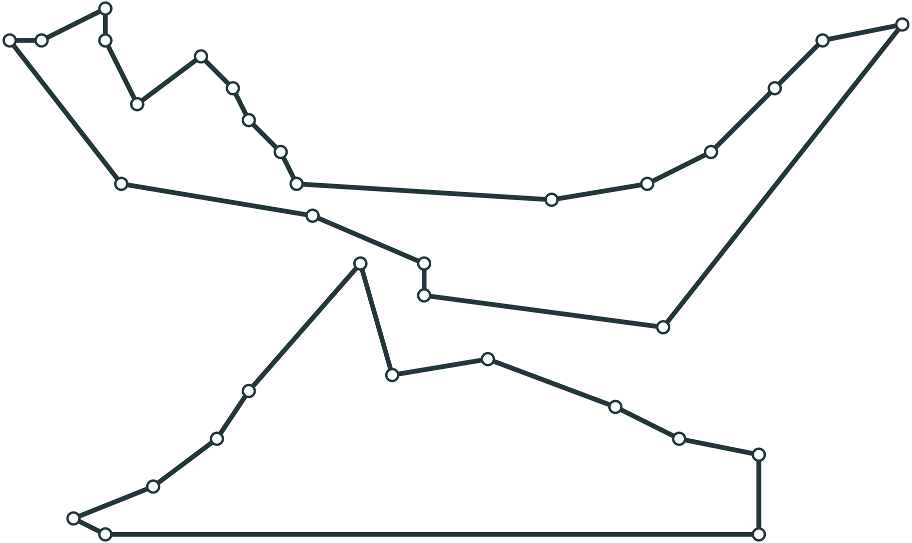
This paper:

$O(1)$-**round**

fully scalable

algorithms!

Embedded planar graphs



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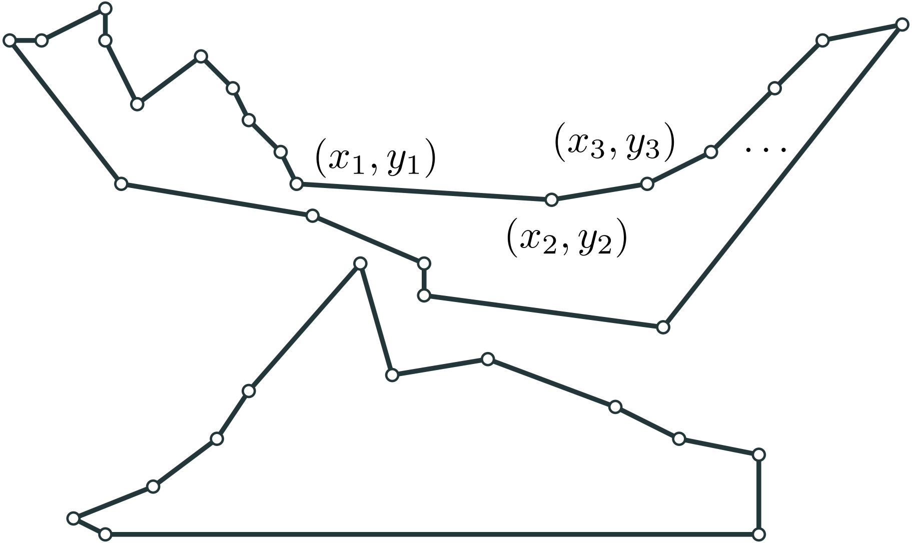

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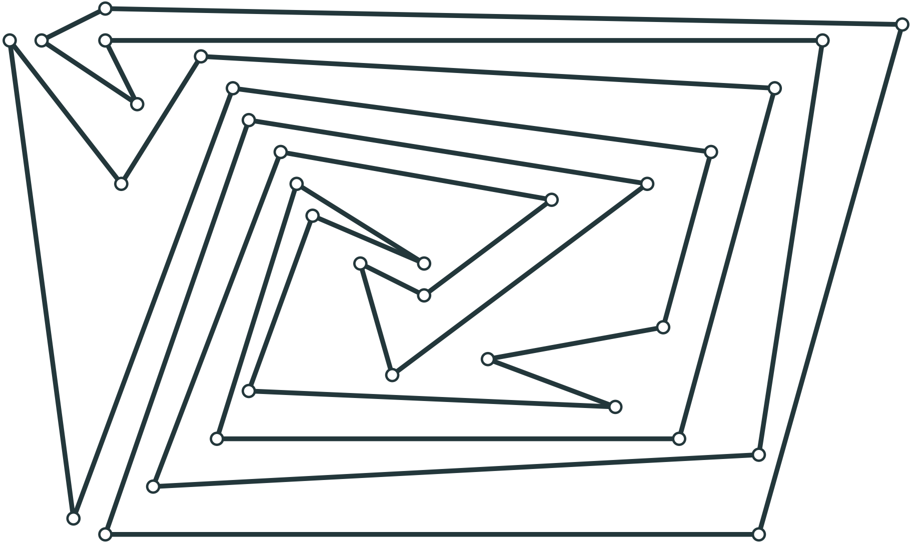

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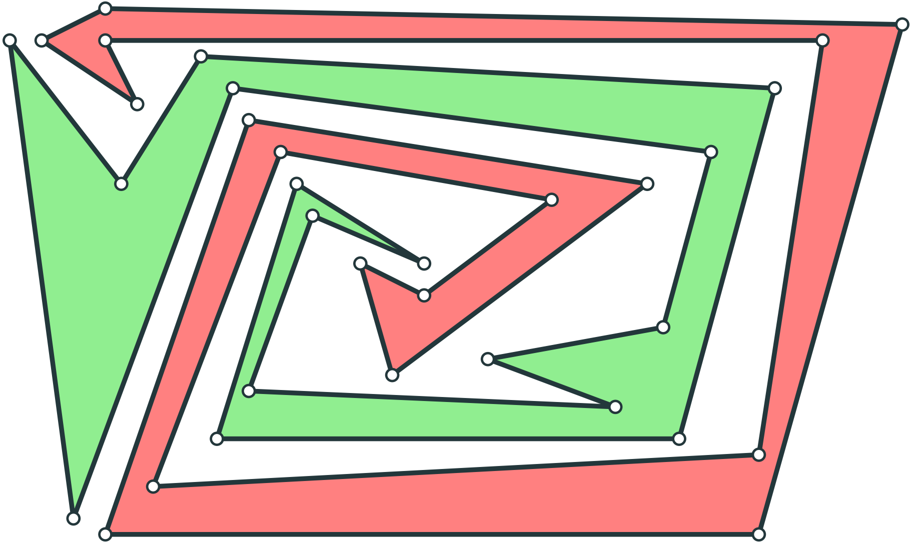

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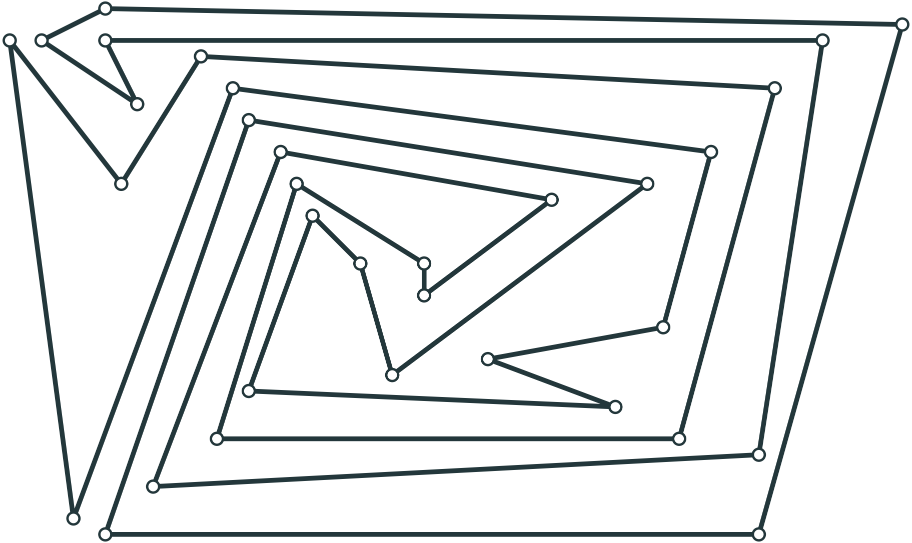

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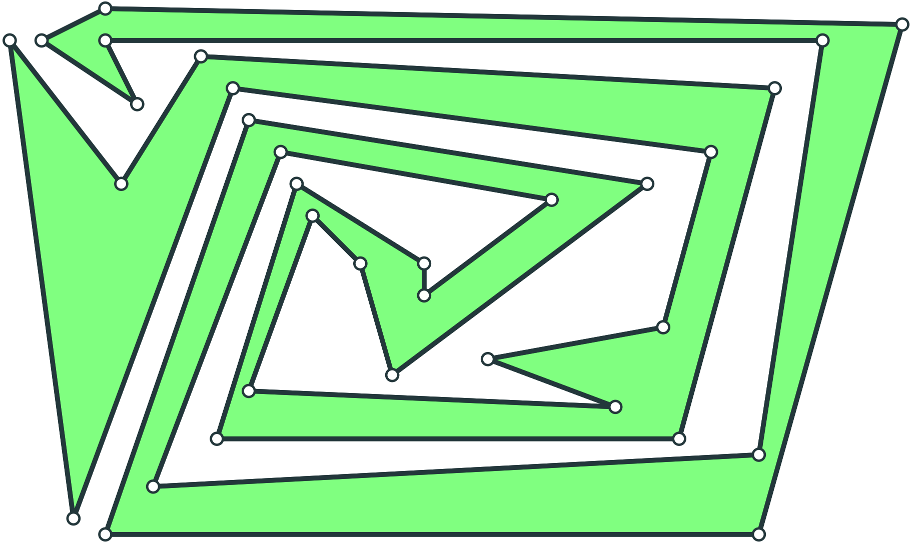

Overcoming the 1-vs-2 cycle conjecture

	Problem	Total space	Memory per machine	Source
Embedded planar graphs	Connected Components	$O(n)$	$n^{2/3+\Omega(1)}$	[HT23]
	Minimum Spanning Tree	$O(n)$	$n^{2/3+\Omega(1)}$	[HT23]
	$O(1)$-approx. SSSP	$O(n)$	$n^{2/3+\Omega(1)}$	[HT23]

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	$(1+\varepsilon)$-approx. SSSP	$O(n)$	n^{δ}	New!

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	$(1+\varepsilon)$-approx. SSSP	$O(n)$	n^{δ}	New!
	$(1+\varepsilon)$-approx. APSP	$O(n^2)$	n^{δ}	New!
	$(1+\varepsilon)$-approx. global min cut	$O(n)$	n^{δ}	New!
	$(1+\varepsilon)$-approx. *st*-max flow	$O(n)$	n^{δ}	New!

Beyond planar graphs

	Problem	Total space	Memory per machine	Source
2D Euclidean MST	$(1+\varepsilon)$-approx.	$O(n)$	n^{δ}	[ANOY14]
	Exact	$O(n)$	$n^{2/3+\Omega(1)}$	[HT23]

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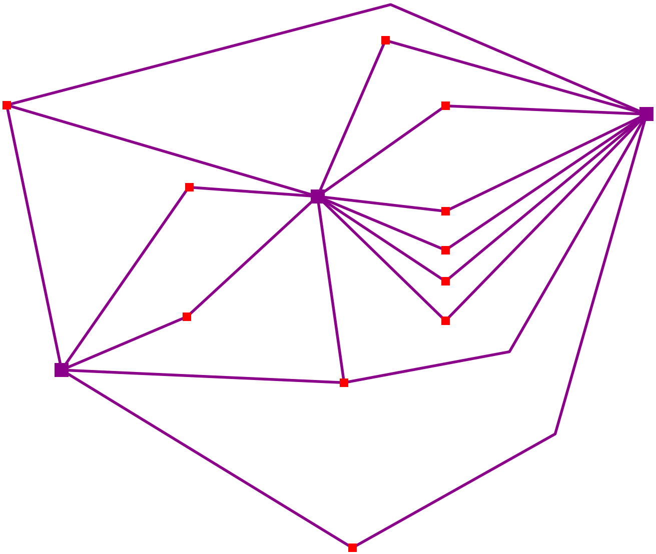
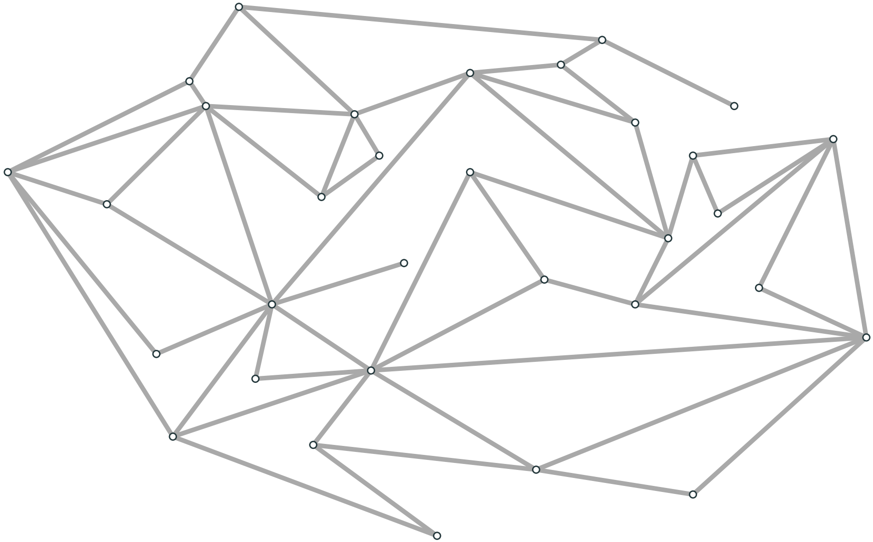
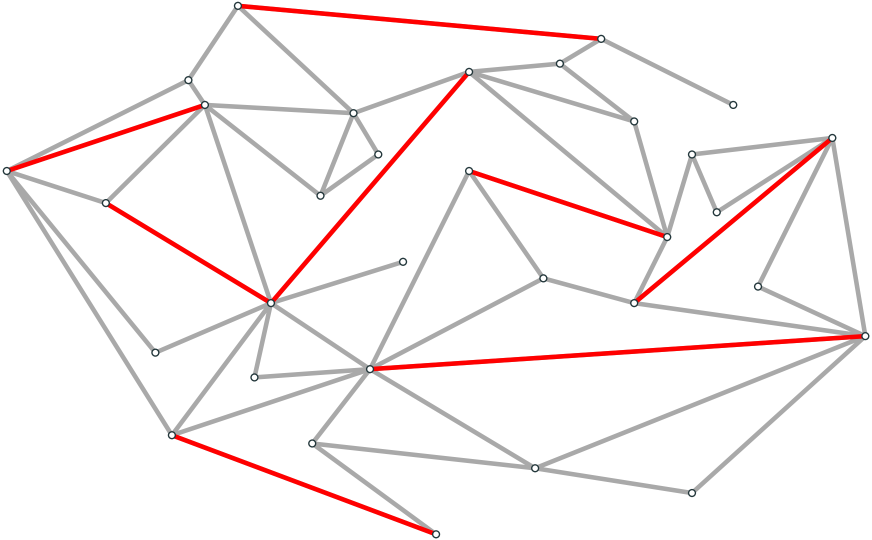
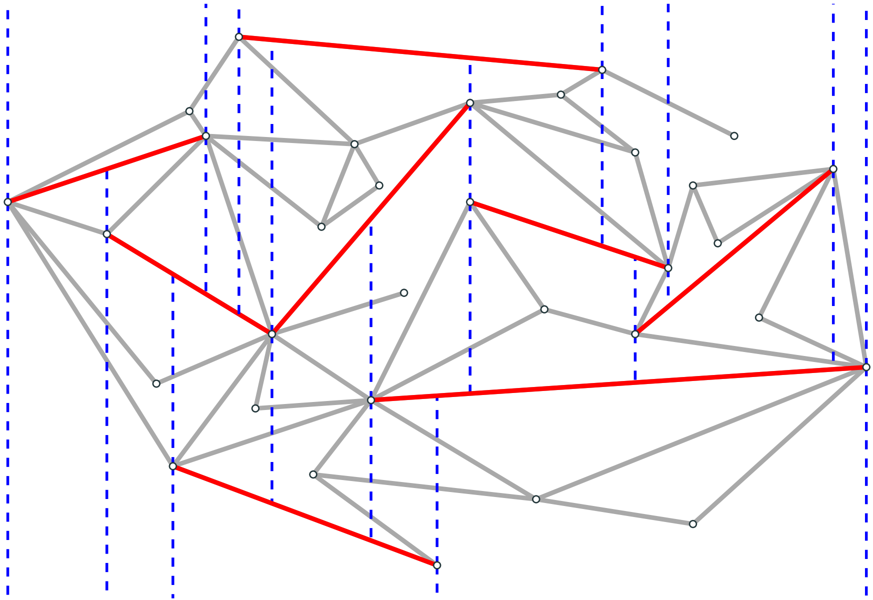
	Exact	$O(n)$	n^{δ}	New!
Edit Distance	$(3+\varepsilon)$-approx.	$\widetilde{O}(n^{(9-4\delta)/5})$	n^{δ}	[BGS21]
	$(1+\varepsilon)$-approx.	$\widetilde{O}(n^{2-\delta})$	n^{δ}	[HSS19]
	$(1+\varepsilon)$-approx. weighted	$\widetilde{O}(n^{2-\delta})$	n^{δ}	New!

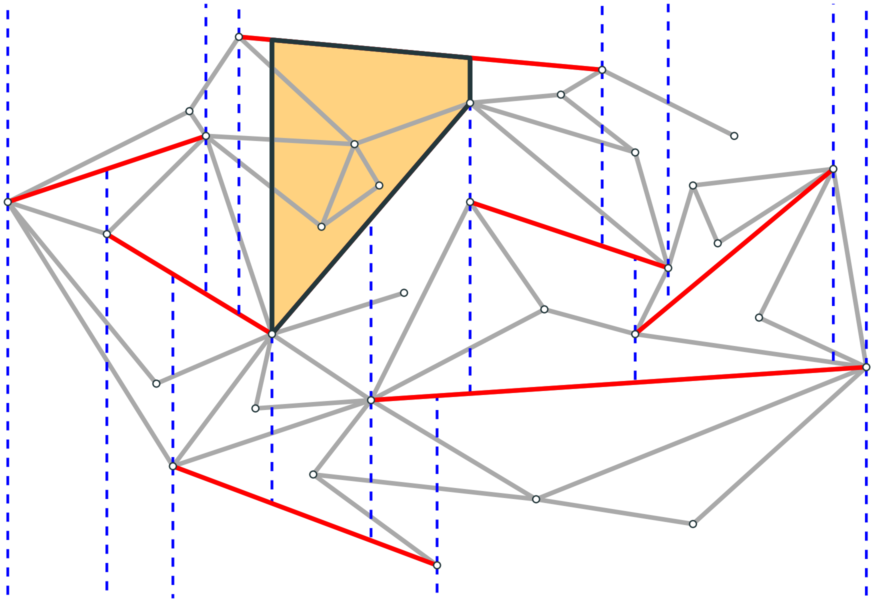
The framework of Holm and Tětek







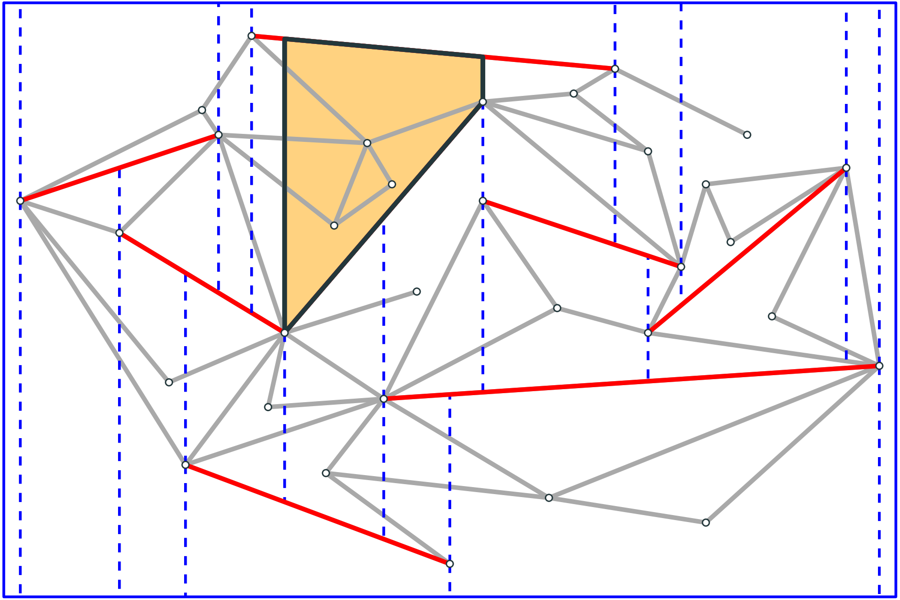




$(1/r)$**-cutting**

$O(r)$ trapezoids

n/r edges / trap.



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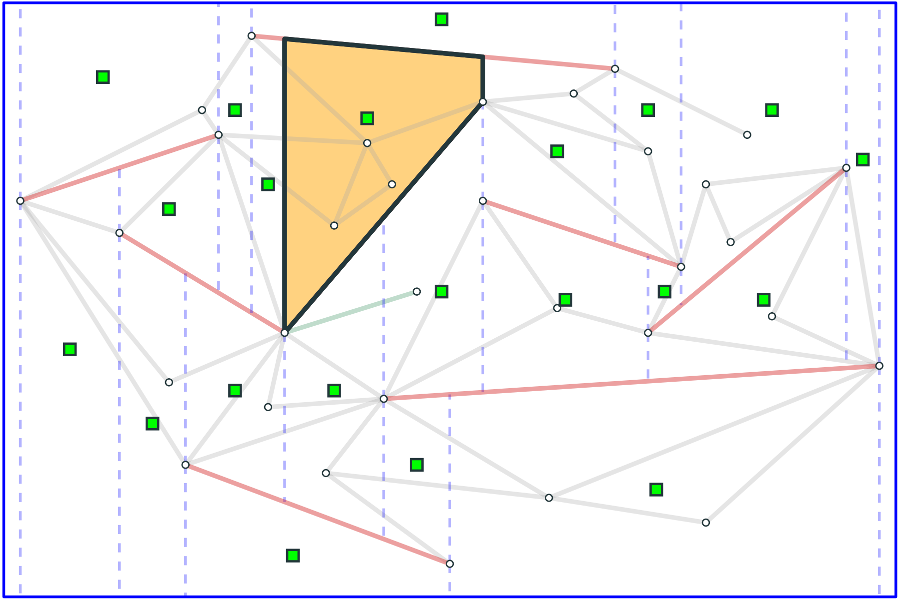

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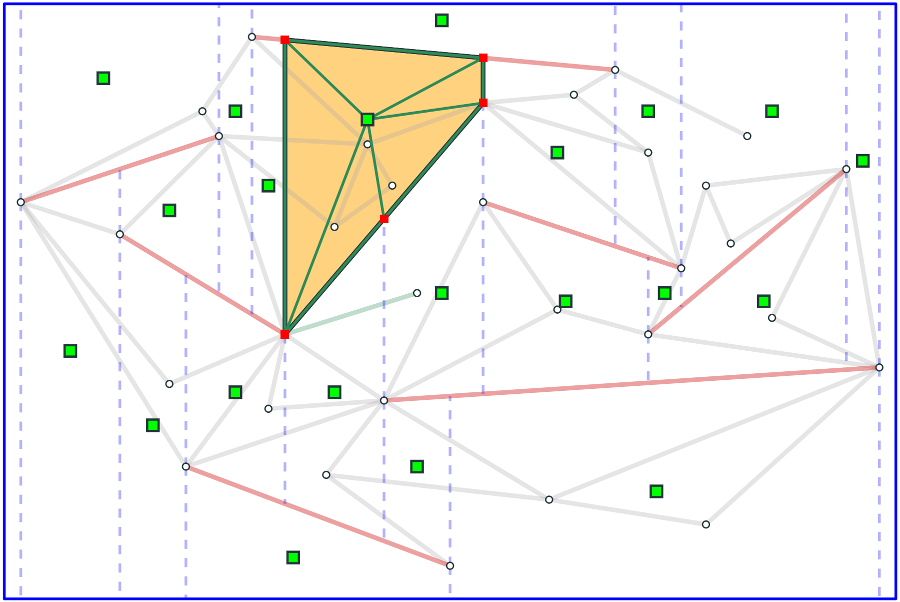

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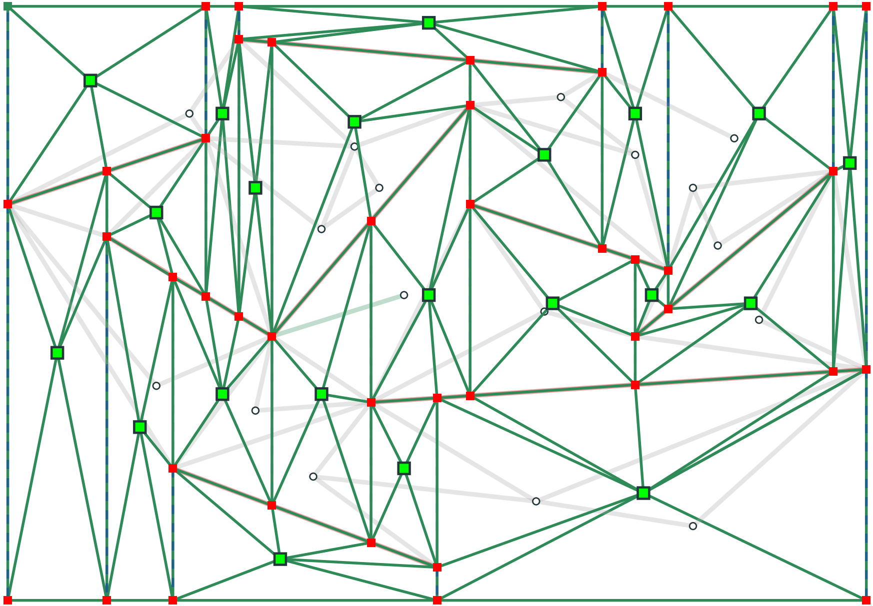

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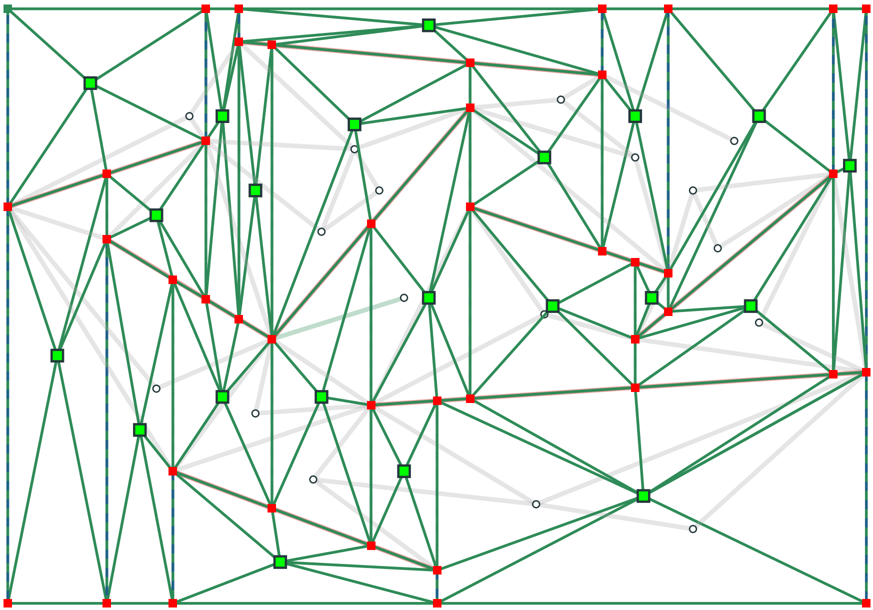

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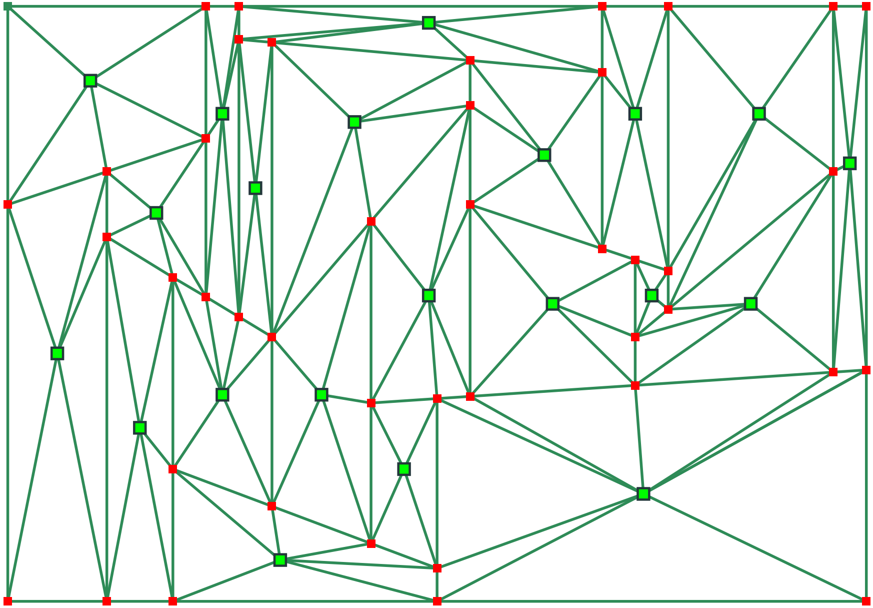
Cutting graph

Planar

$O(r)$ vert. and edges



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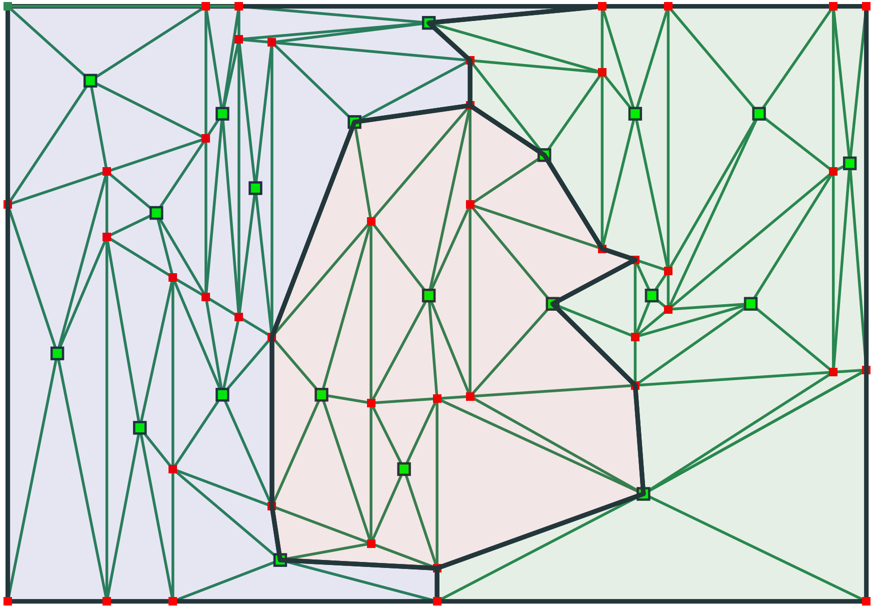

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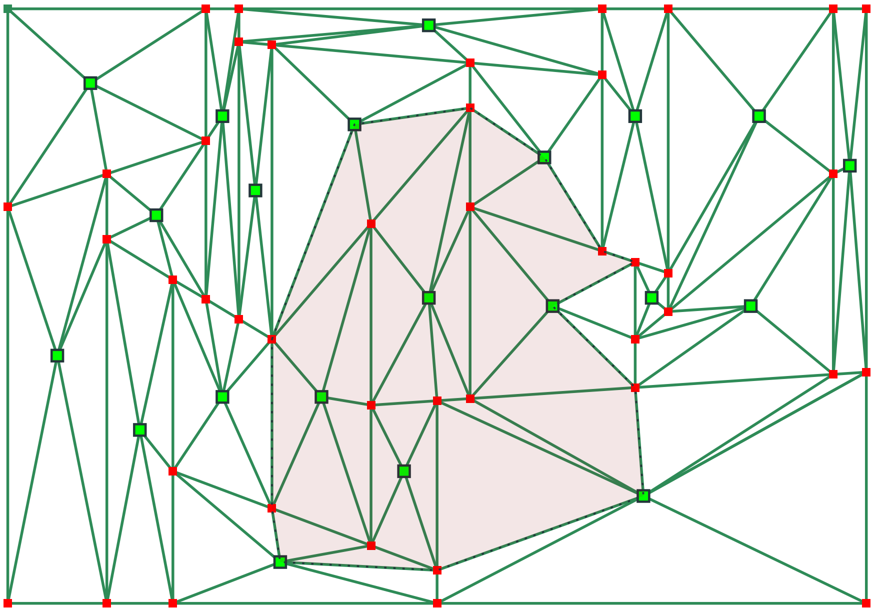
Planar s-divisions

$O(r/s)$ regions

$O(s)$ vert. / region

$O(\sqrt{s})$ vert. / bound.

$O(1)$ holes



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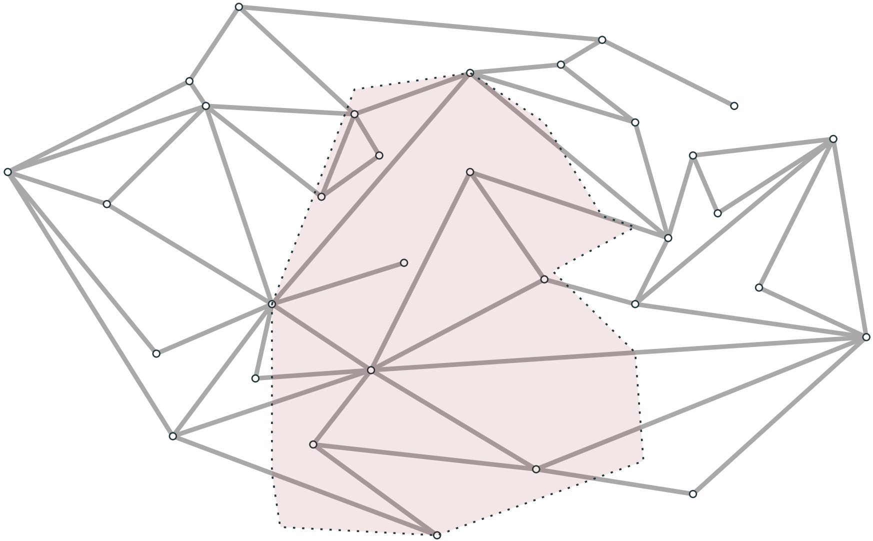

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Cutting-division

$O(r/s)$ regions

$O\left(\frac{ns}{r}\right)$ edges / region

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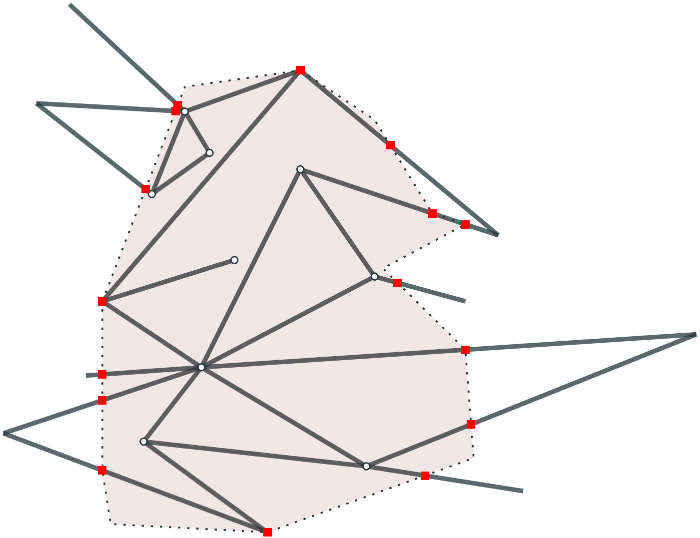

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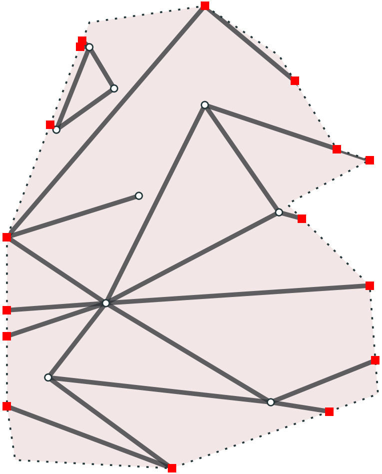

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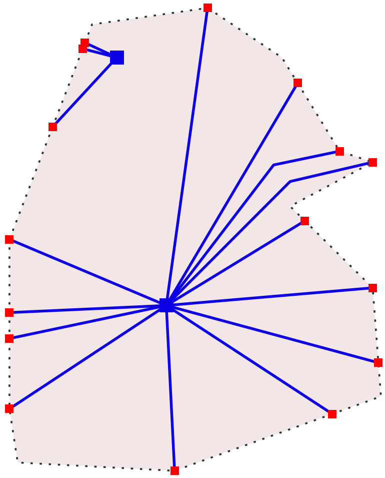

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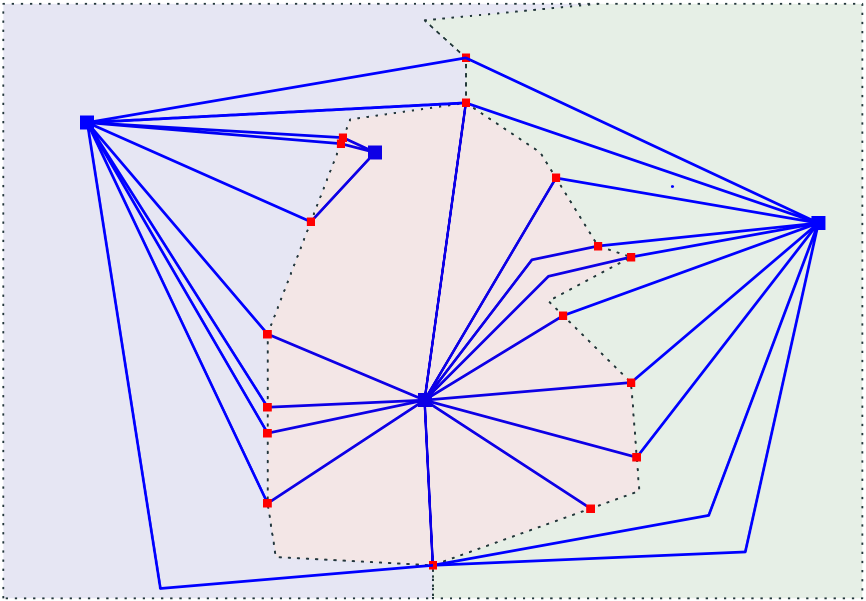

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Picking the parameters - Take 1 [Holm Tětek '23]

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- Local space usage for $(1/r)$-cutting: $O(r)$.

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Space usage: $O(ns/r + n/\sqrt{s} + r)$. Choose $s = r^{2/3}$ and $r = n^{1/2}$ for $\mathcal{S} = O(n^{3/4})$.

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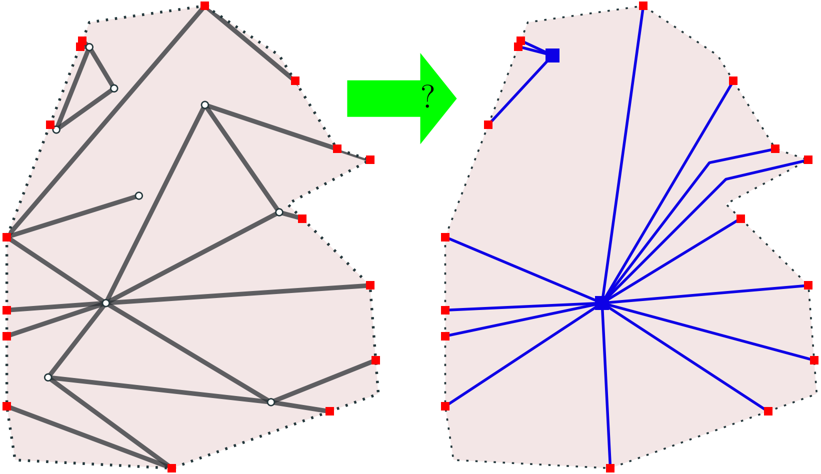
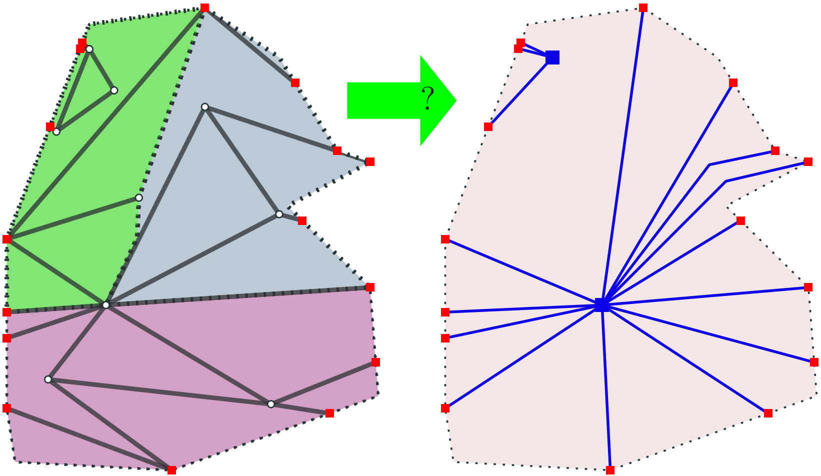
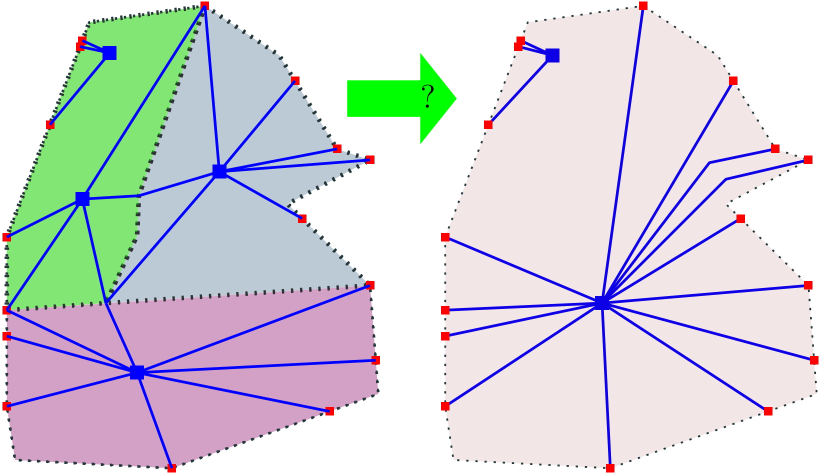
Can improve to $S = n^{2/3+\Omega(1)}$ by using some recursion.

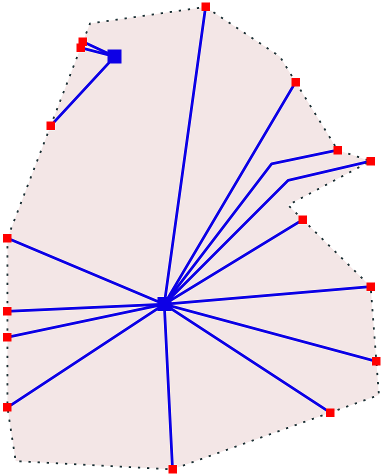
New framework:

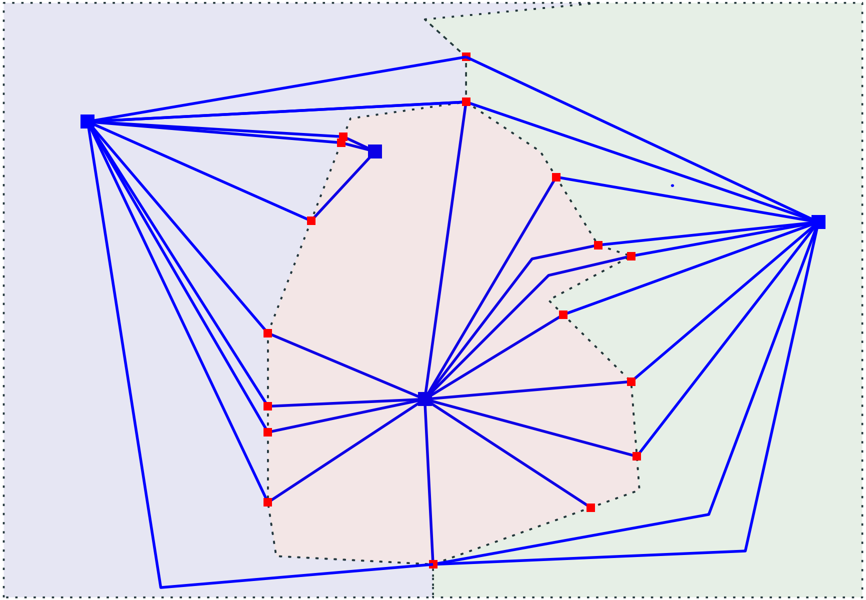
New framework:
More recursion + graph drawing!

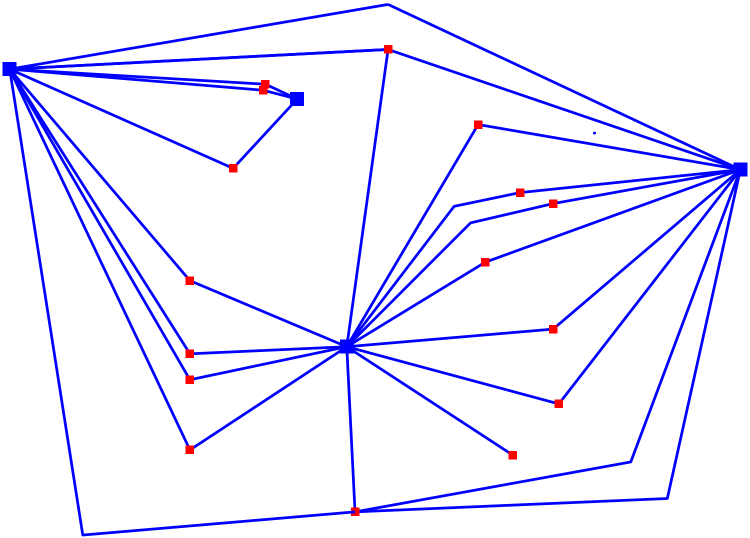


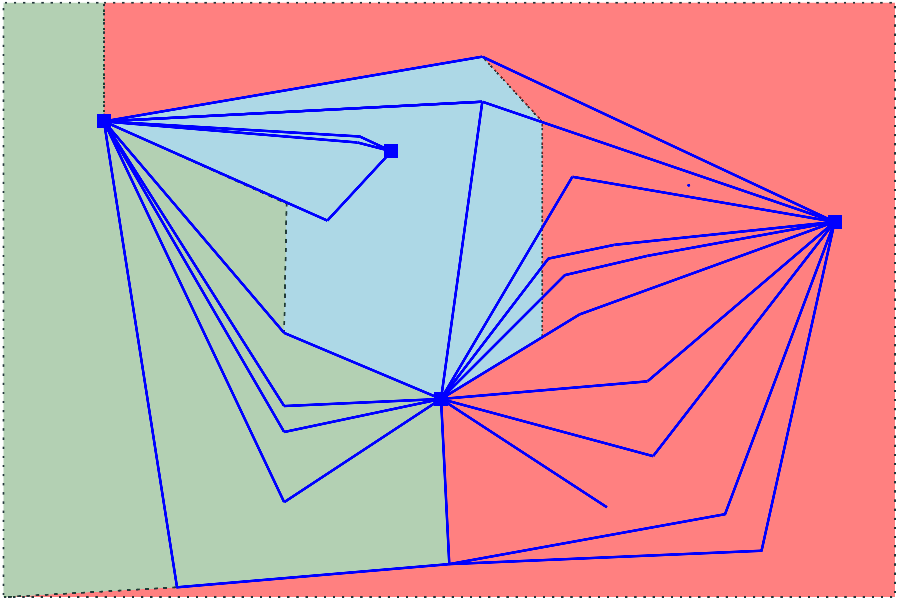


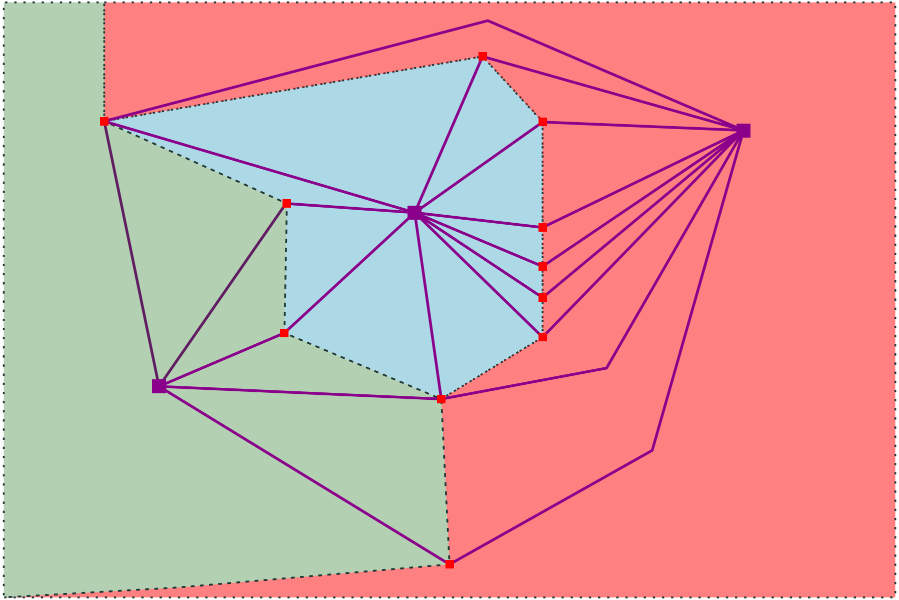


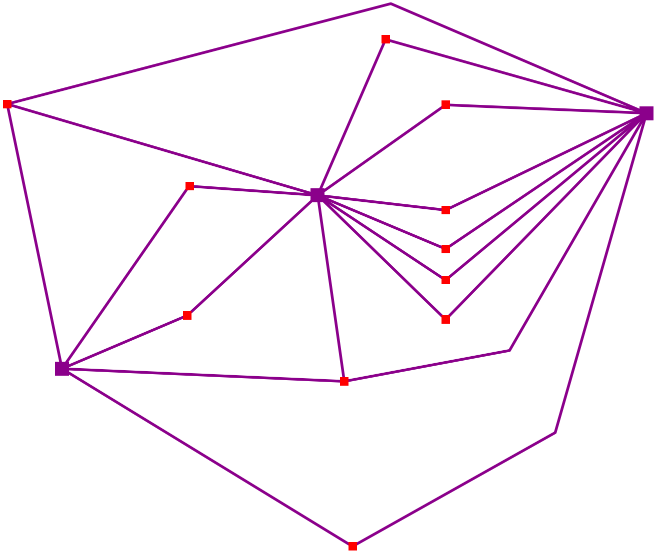












Picking the parameters - Take 2 ($\mathcal{S} = n^\delta$)

$O(r/s)$ regions $O\left(\frac{ns}{r}\right)$ edges / region $O\left(\frac{n\sqrt{s}}{r}\right)$ edges / boundary

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Graph with $O(n)$ edges $\rightarrow$ Graph with $O(n^{1-\delta/3})$ edges.

Iterate this until graph is small!

Technical challenges

$O(n^{\delta/3})$ regions $O\left(n^{1-\delta/3}\right)$ edges / region $O\left(n^{1-2\delta/3}\right)$ edges / boundary

Technical Challenges

- The number of boundary edges is large $O(n^{1-2\delta/3})$

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- Base case?
  - Solution: Graph drawing!
- Precision issues?

Technical issues (cont.)

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The boundary of a region is a complex polygon of size $O(n^{\delta/3})$ with $O(1)$ holes

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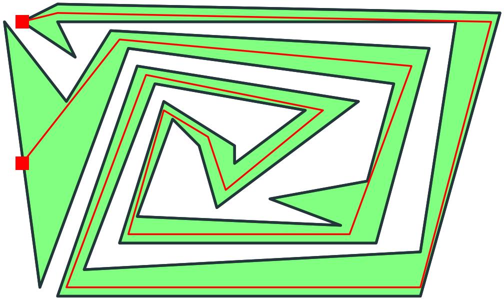

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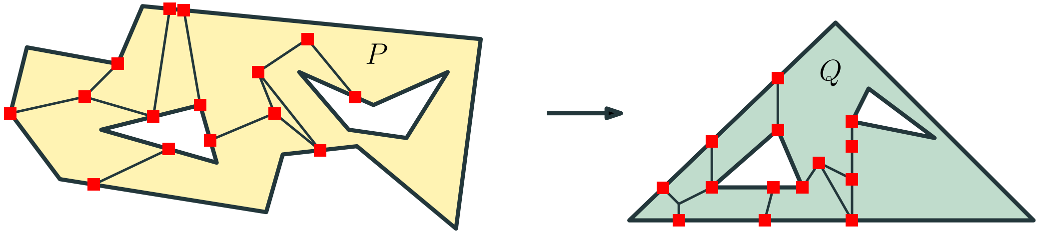
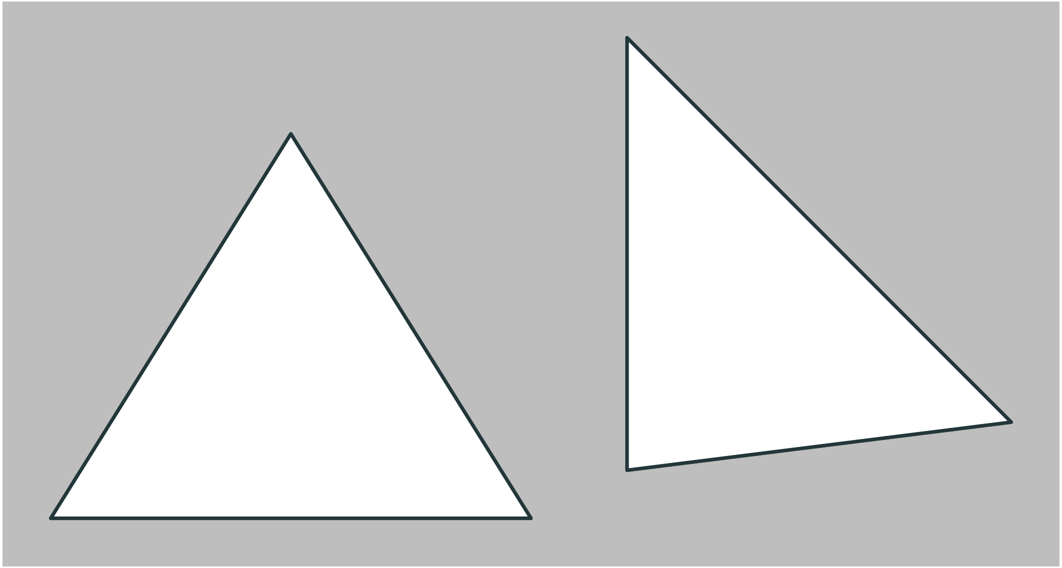


Advantages of triangular boundaries

Compatible triangulations



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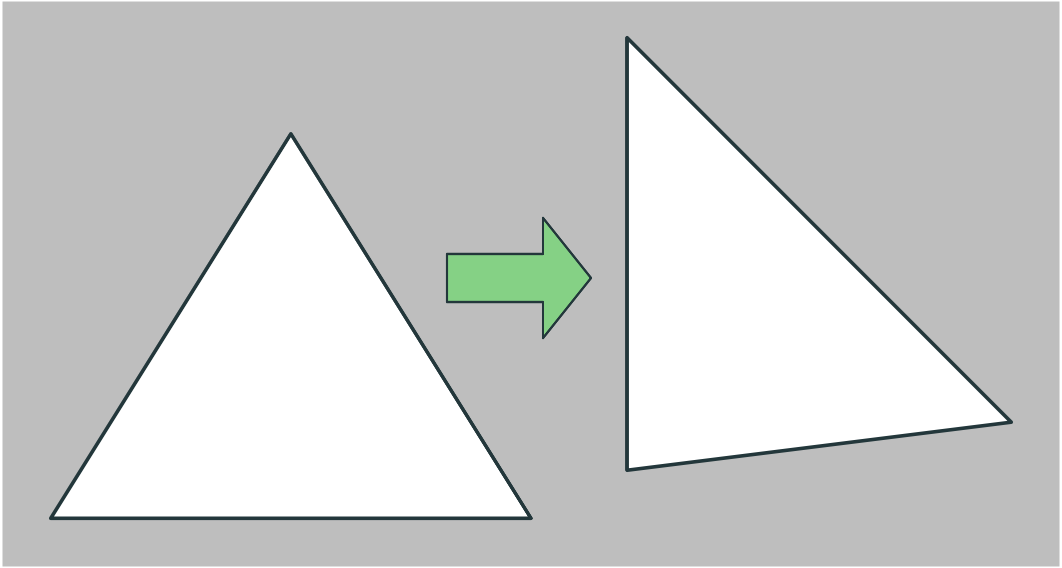

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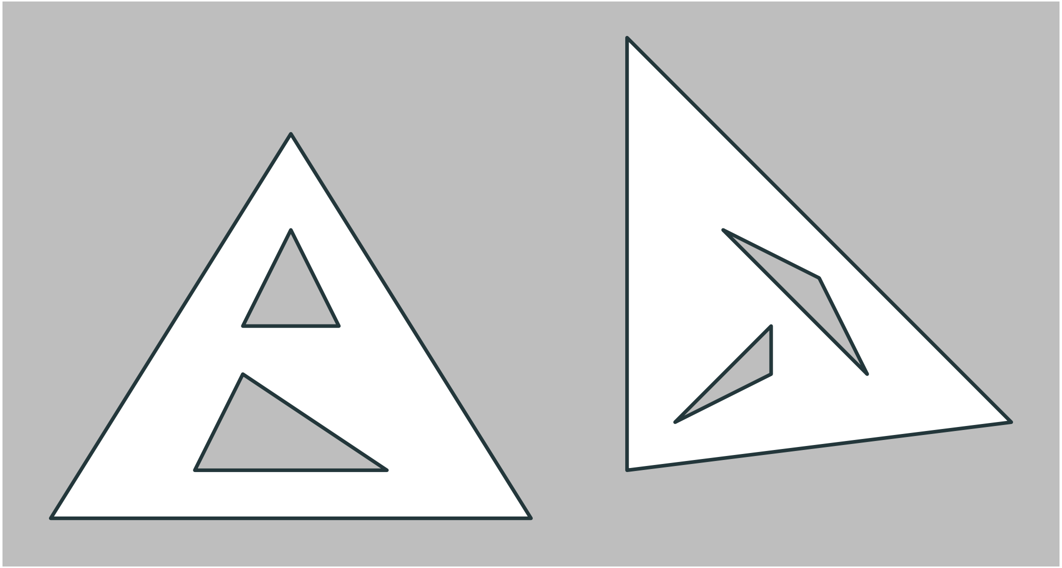

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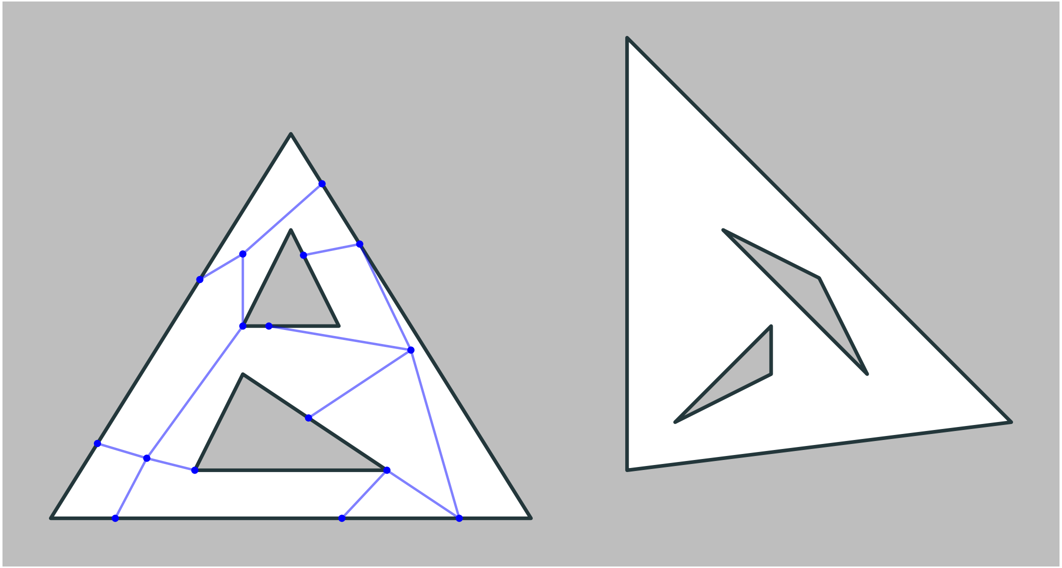

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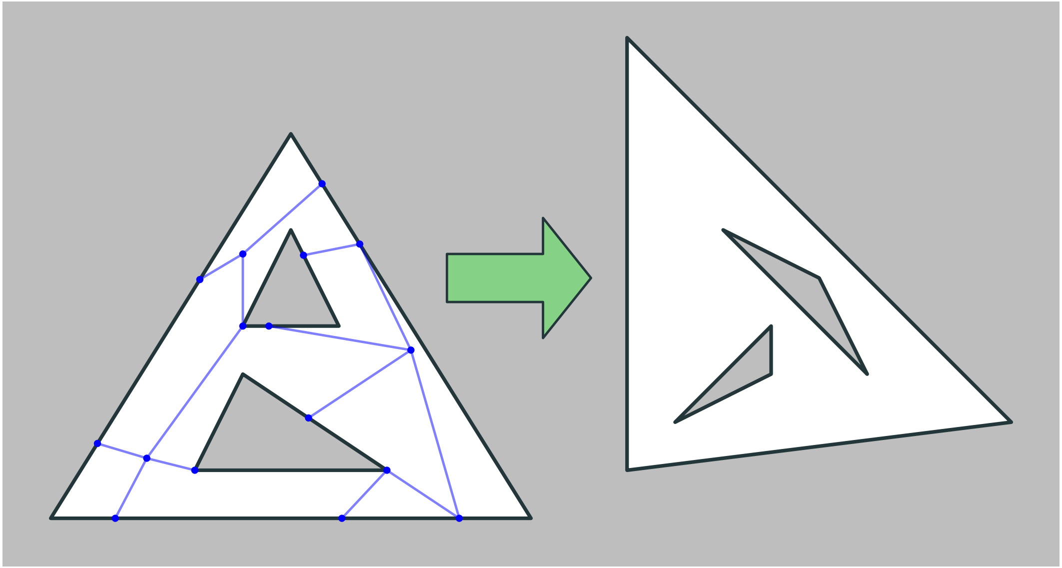

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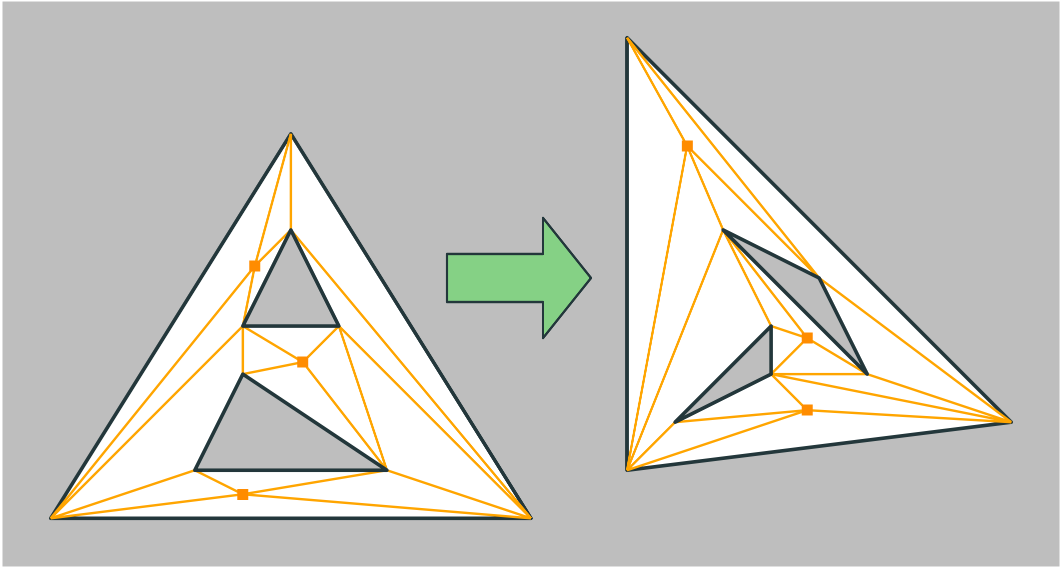

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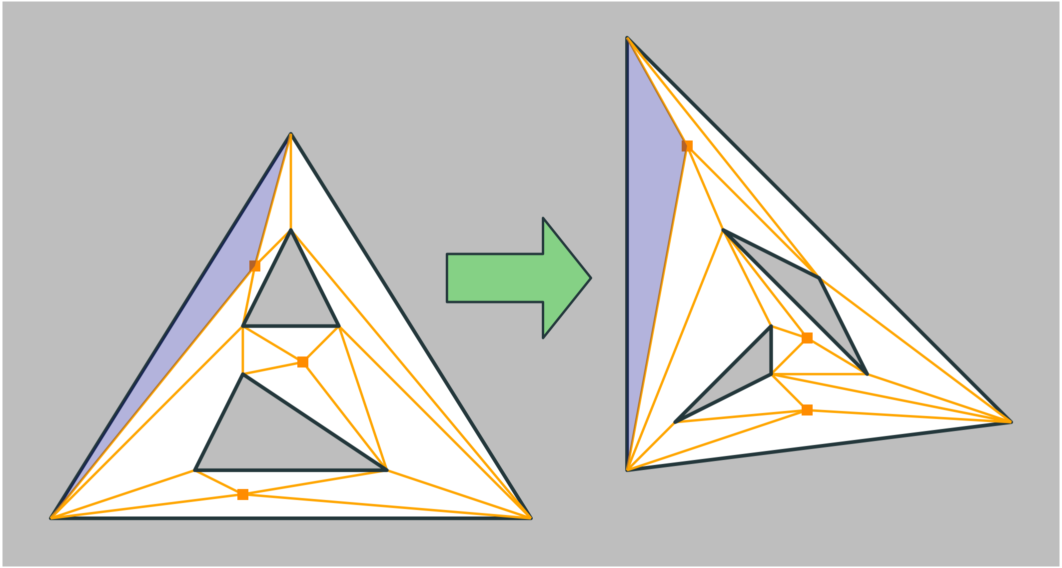

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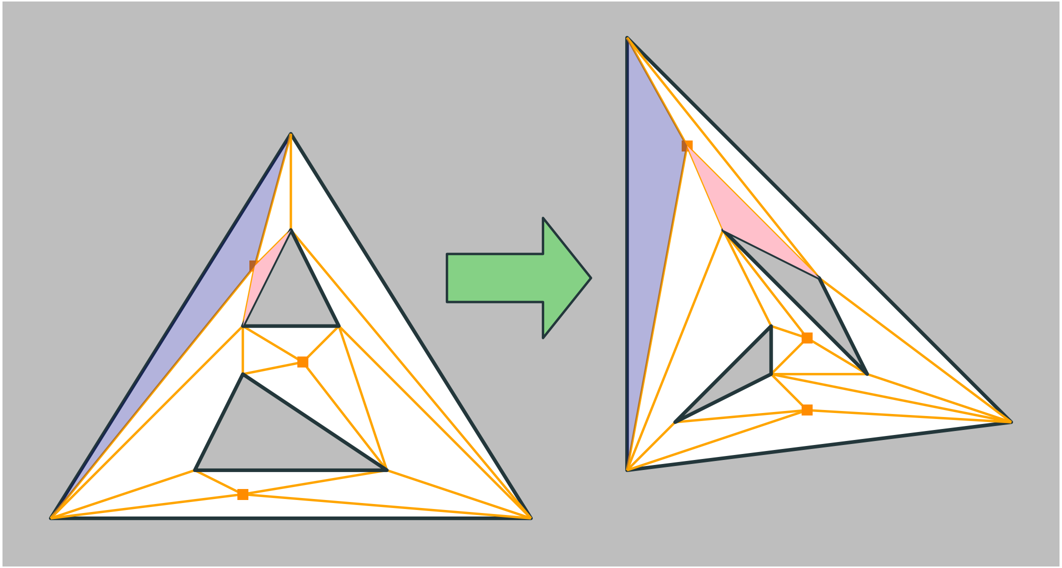

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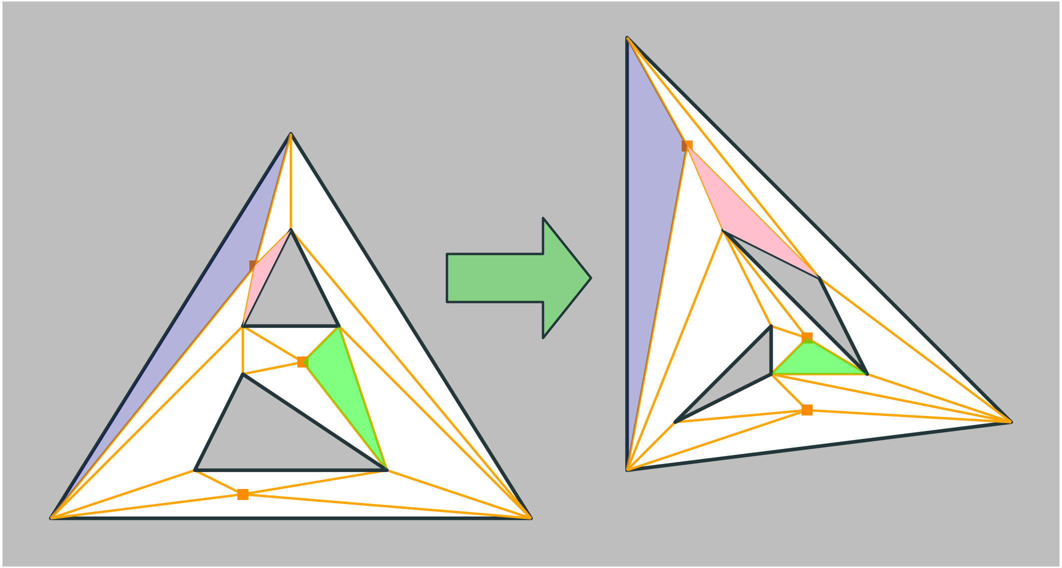

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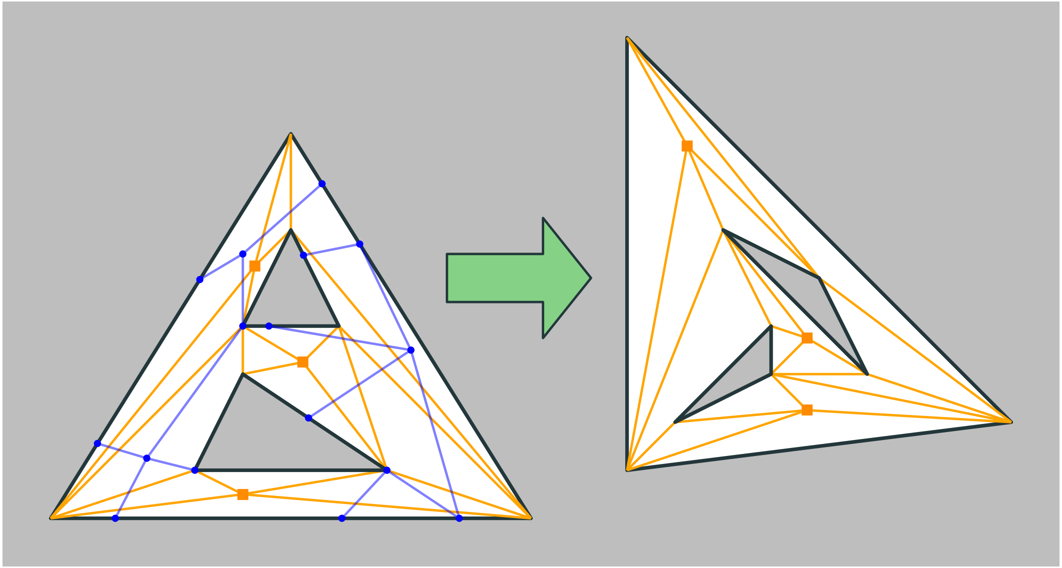

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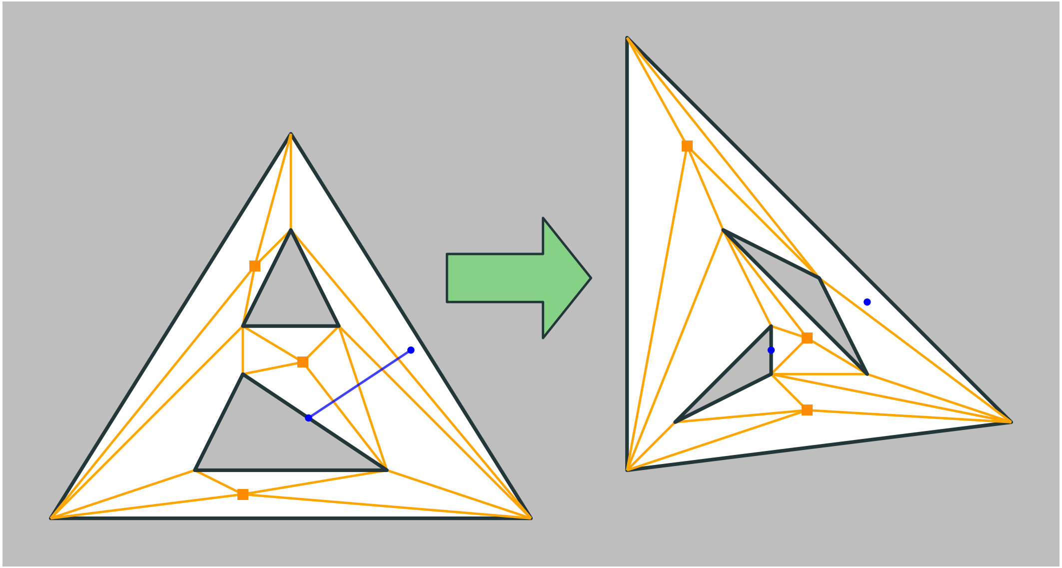

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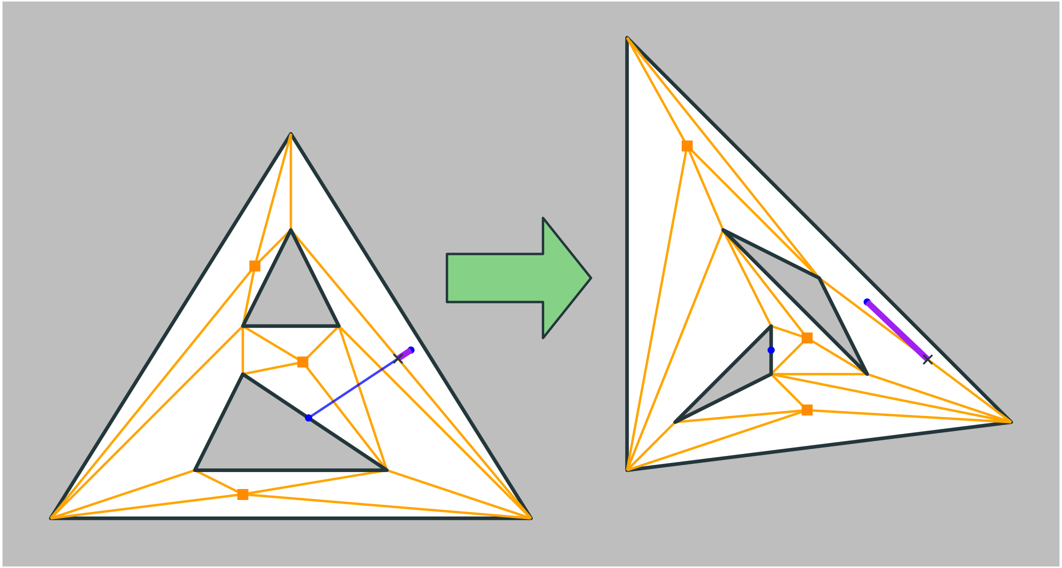

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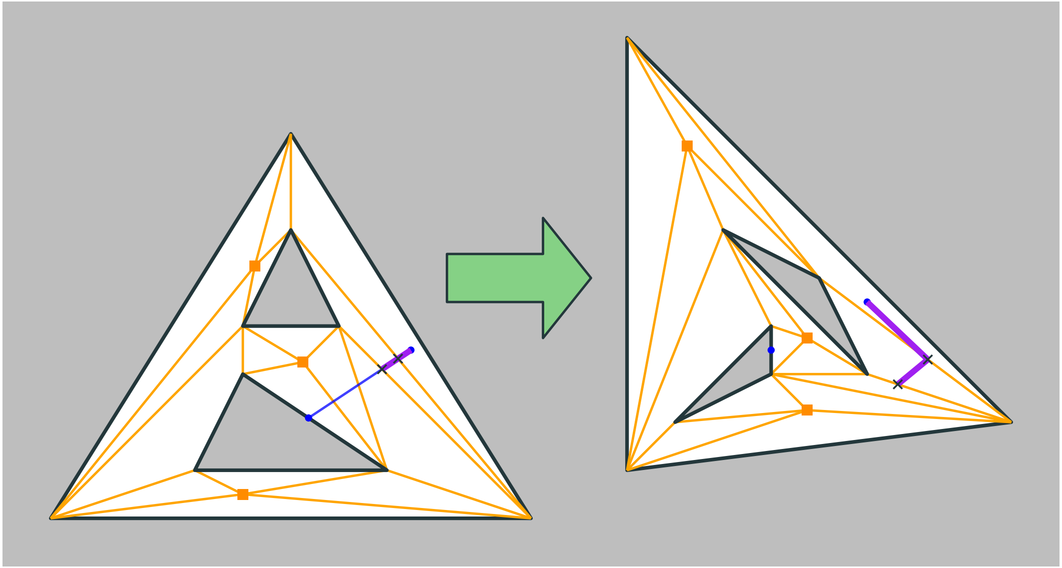

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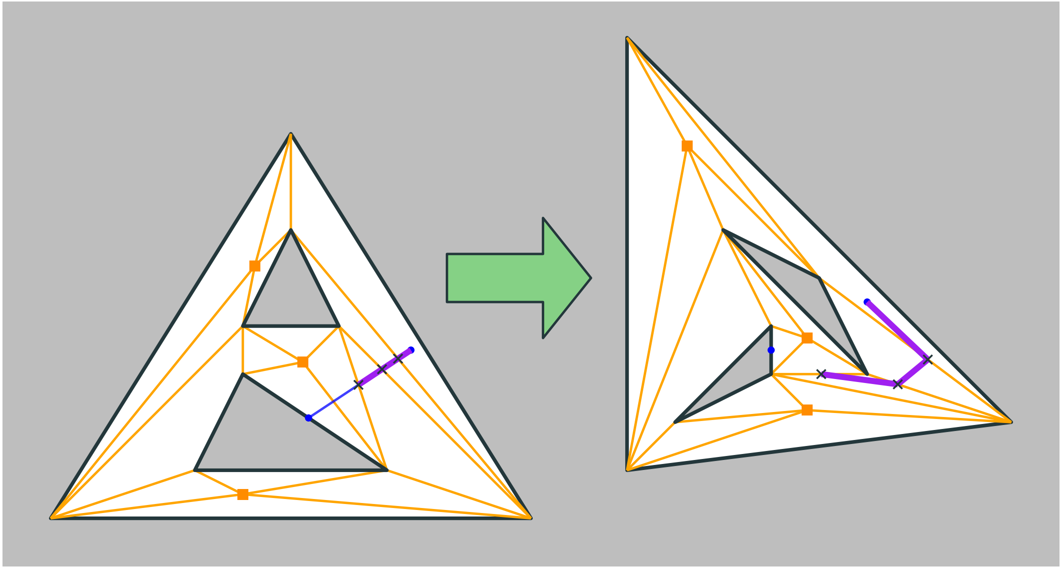

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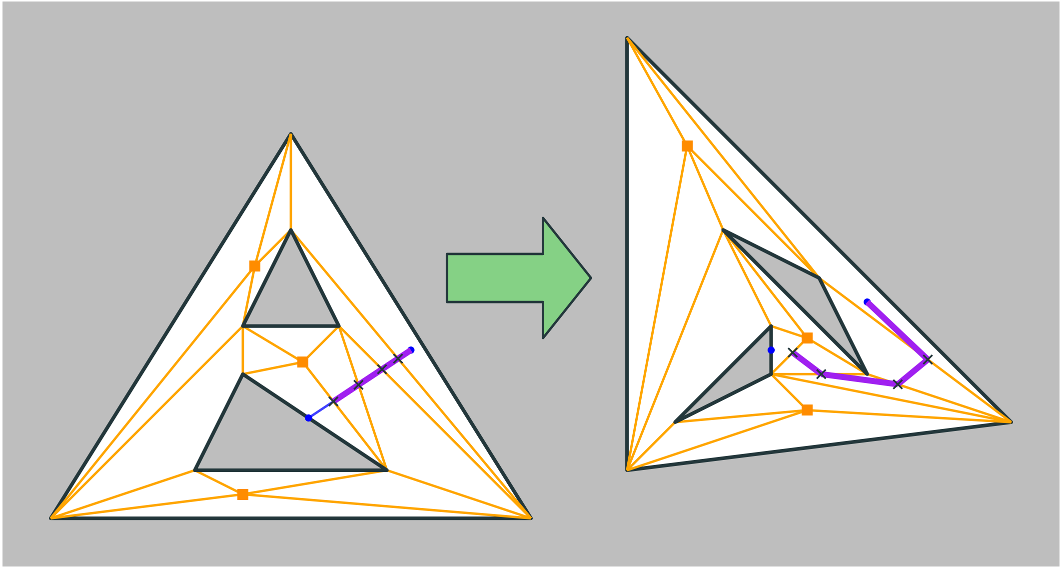

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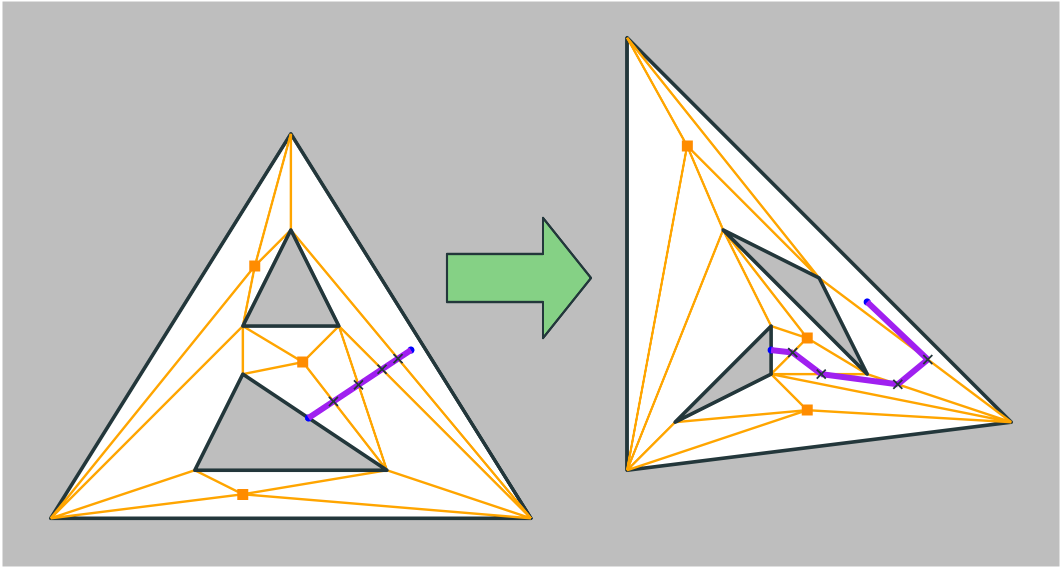
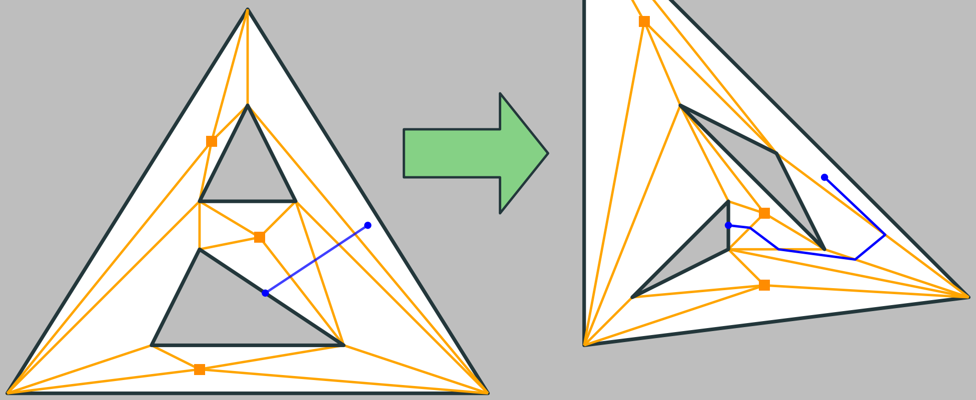


Compatible triangulations

h holes $\Rightarrow O(h^2)$ bends



Technical issues in the gluing step (cont.)

We need to join these triangles together.

Subdivision may have nested holes (can be nested $O(n^{\delta/3})$ deep).

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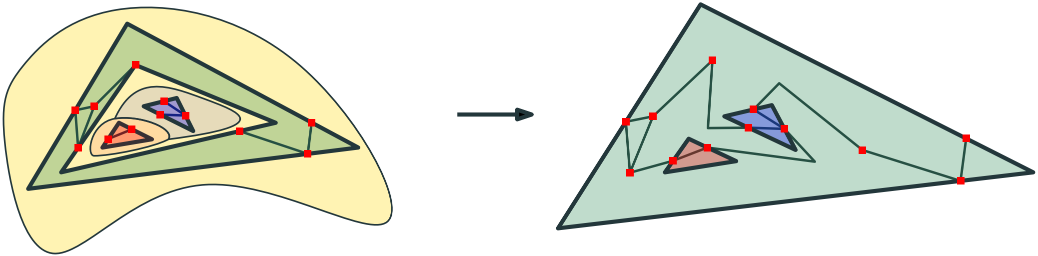

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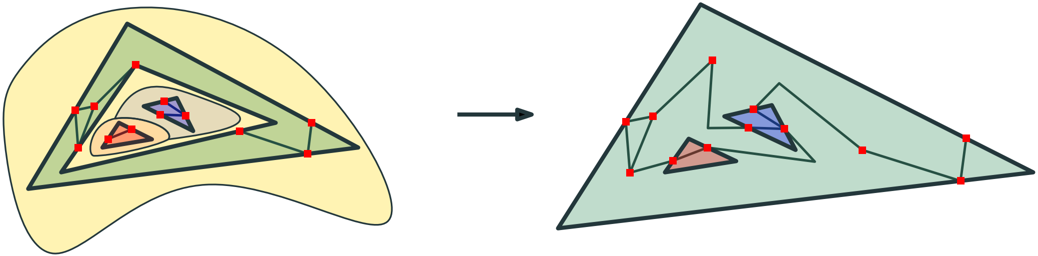

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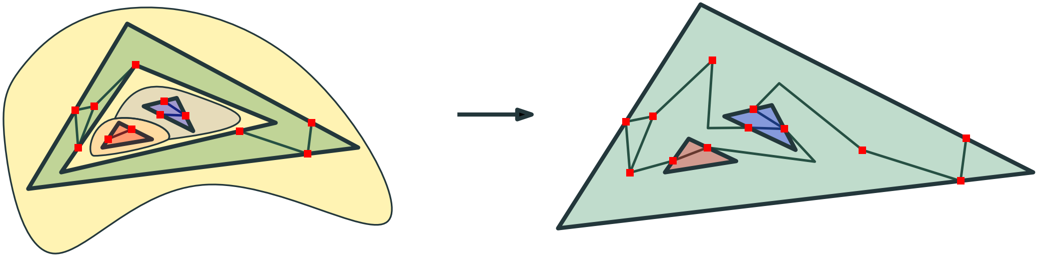
- Solution: Parallel routing technique + a **scaffold graph**.



Routing between curves in parallel

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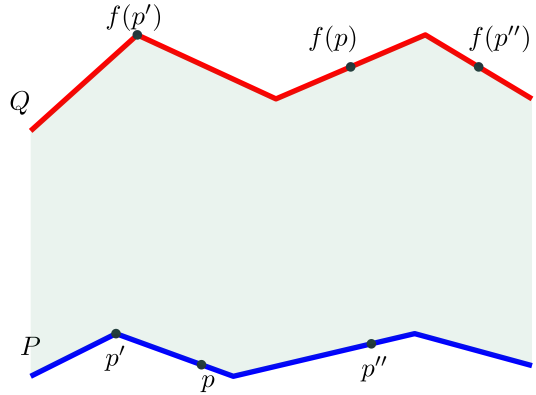

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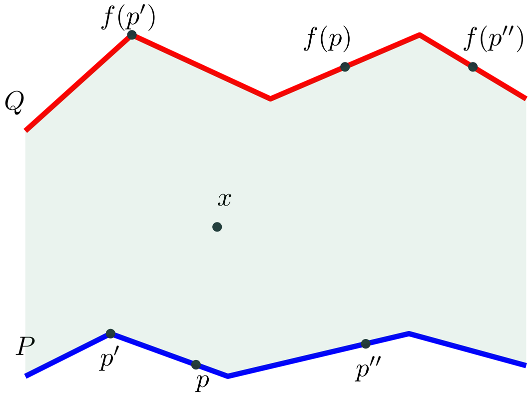

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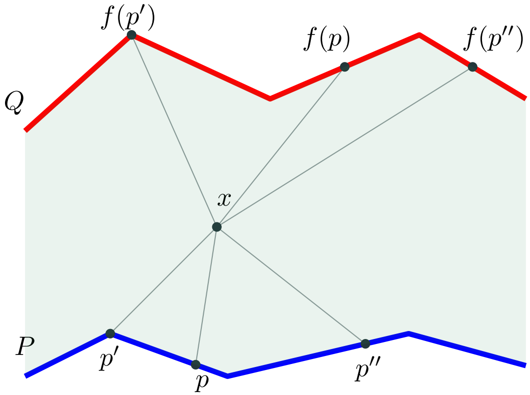

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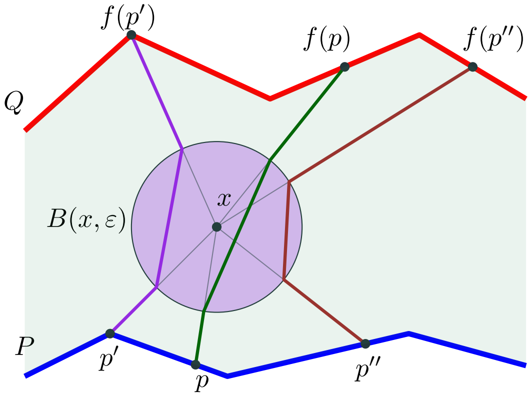

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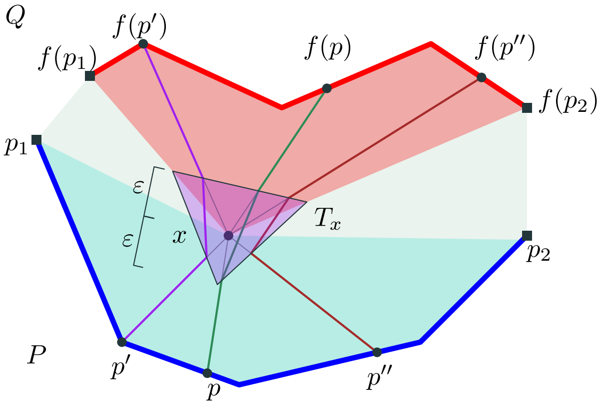
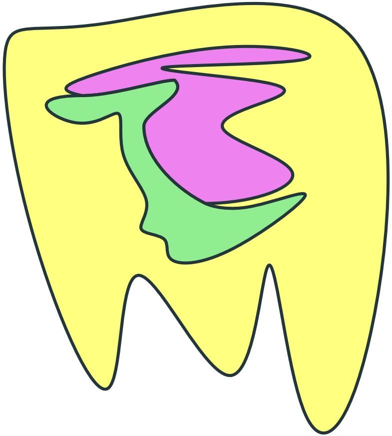


Scaffold construction

Scaffold construction (First attempt)



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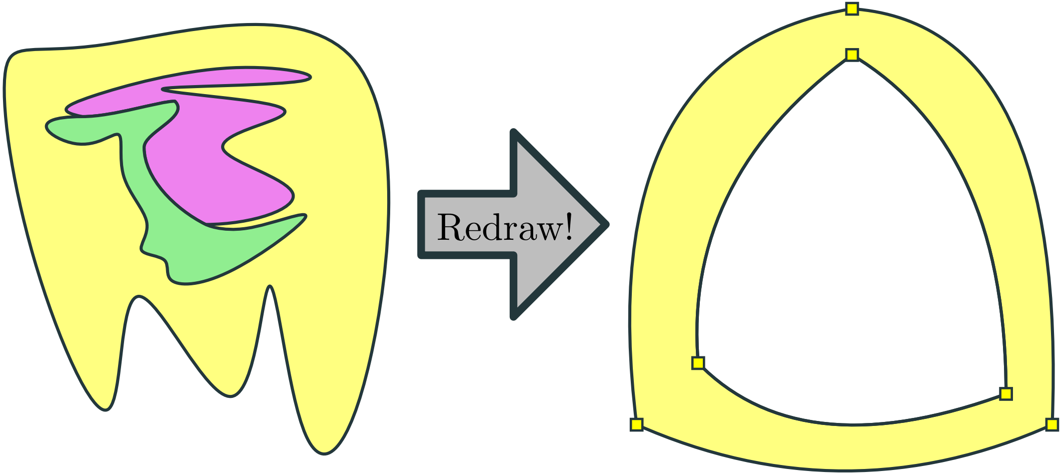

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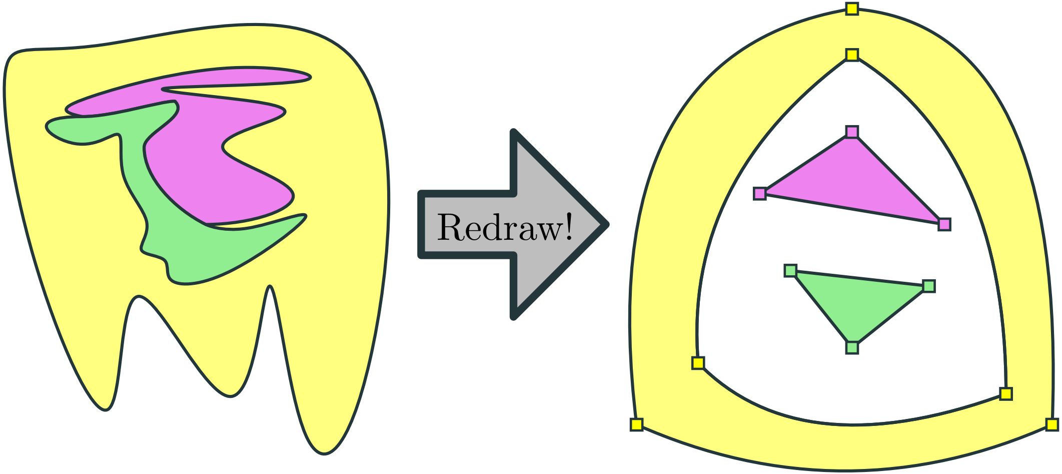

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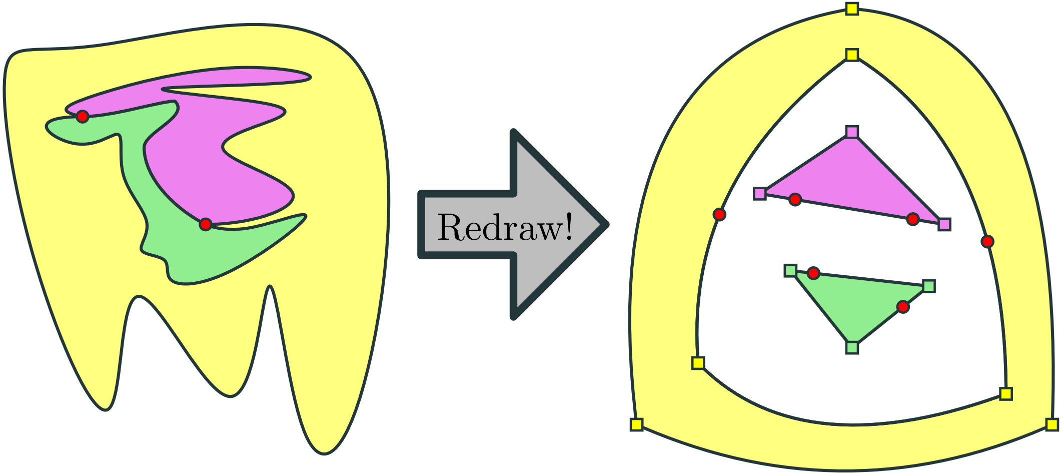

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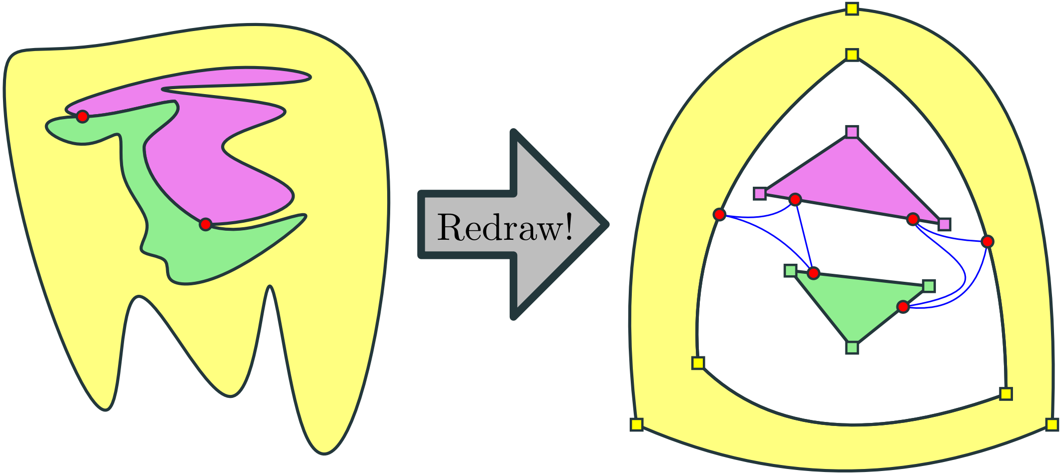

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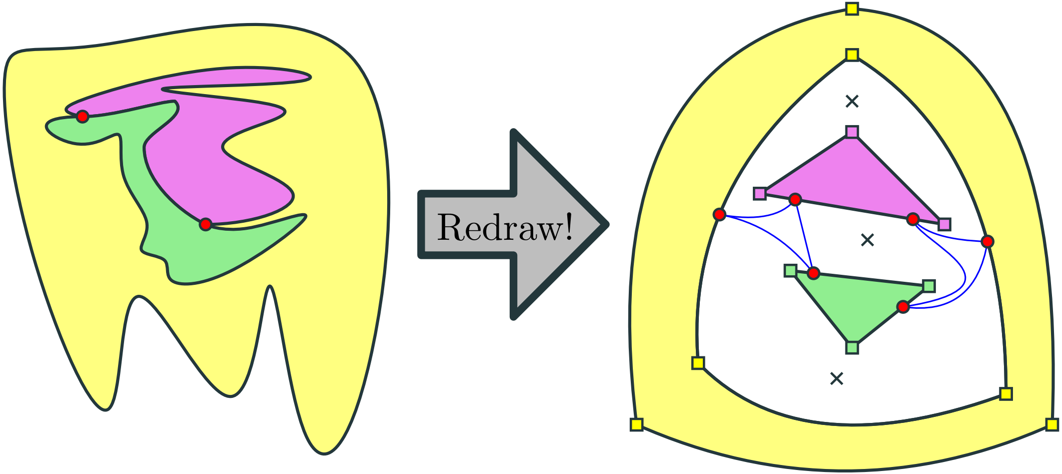

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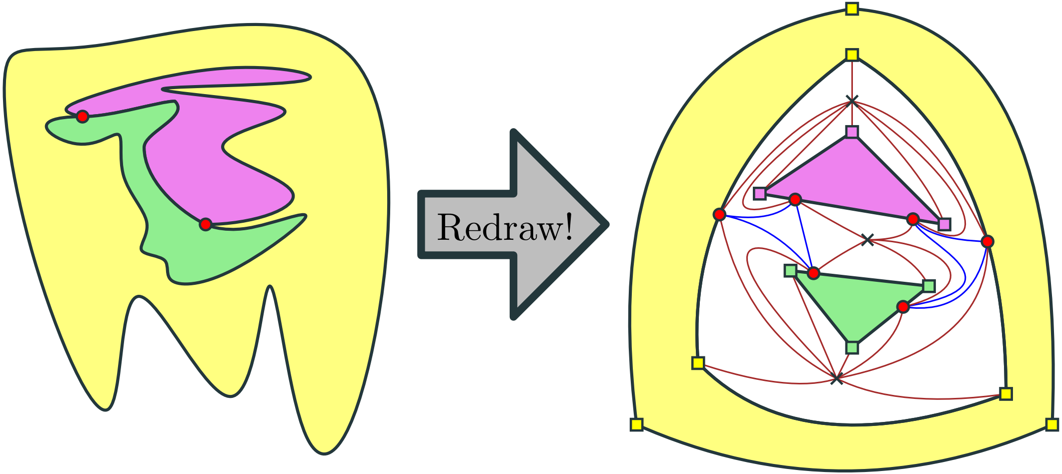
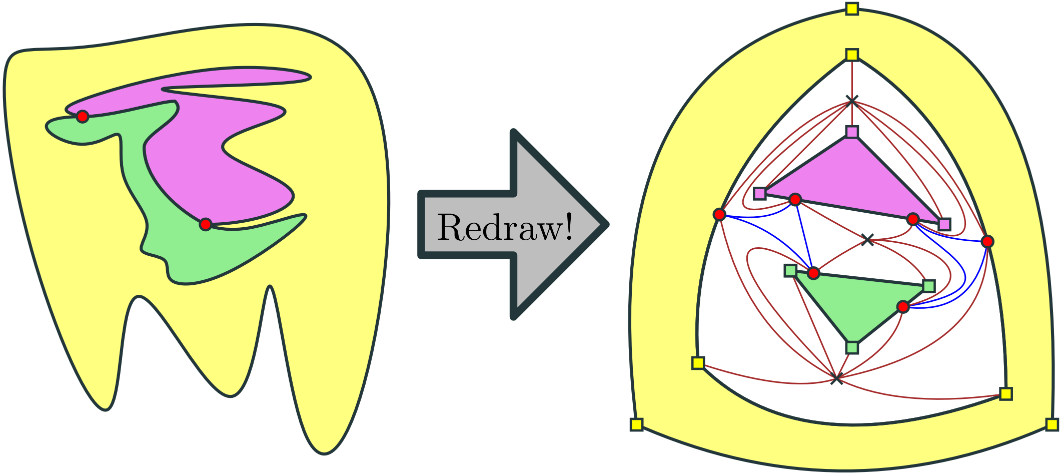


Scaffold construction (First attempt)

The scaffold has size $O(n^{\delta/3}) < \mathcal{S}$ so can be stored (and drawn) by all machines!



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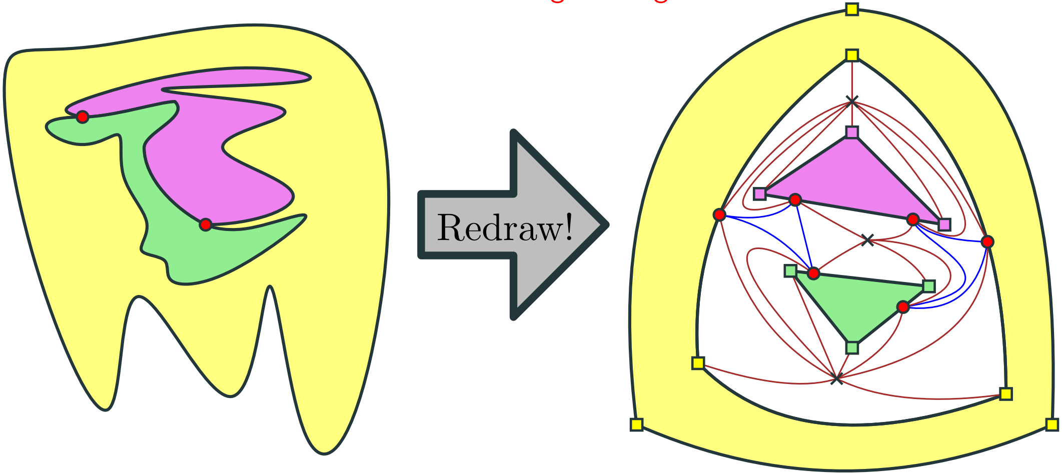
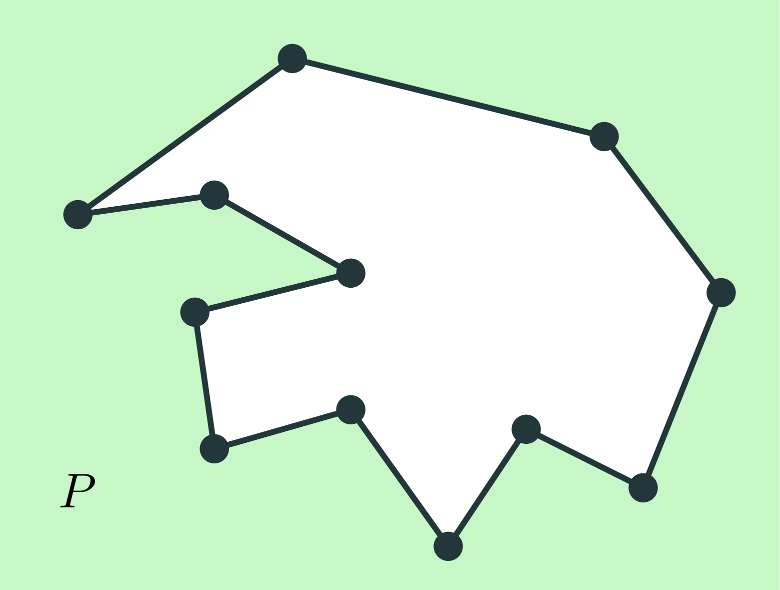
Faces are no longer triangles!



Scaffold construction for a polygon



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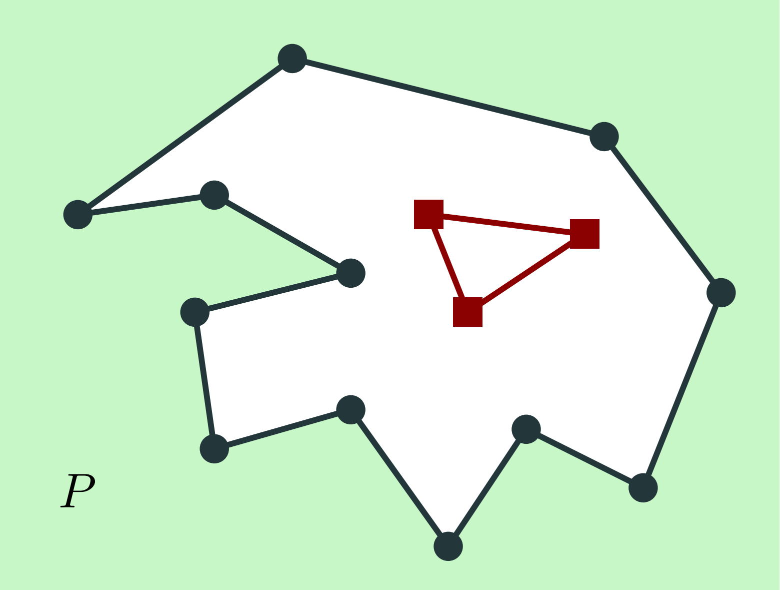

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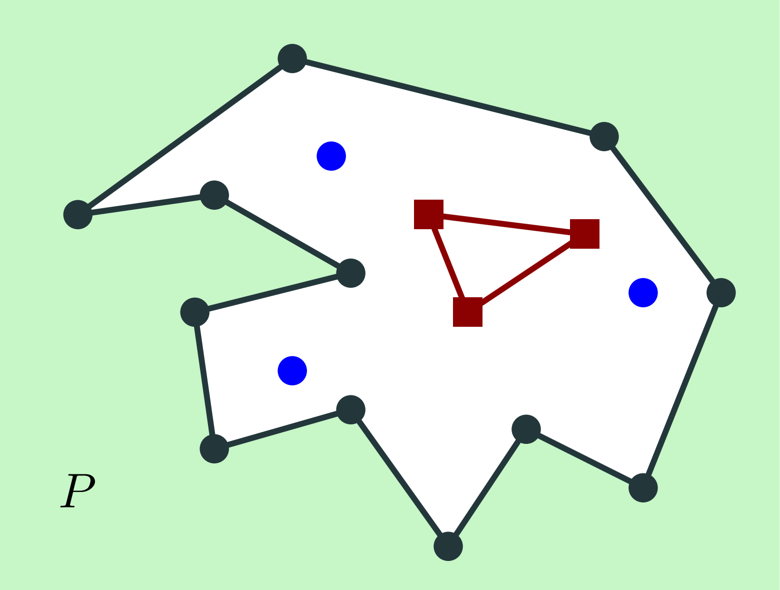

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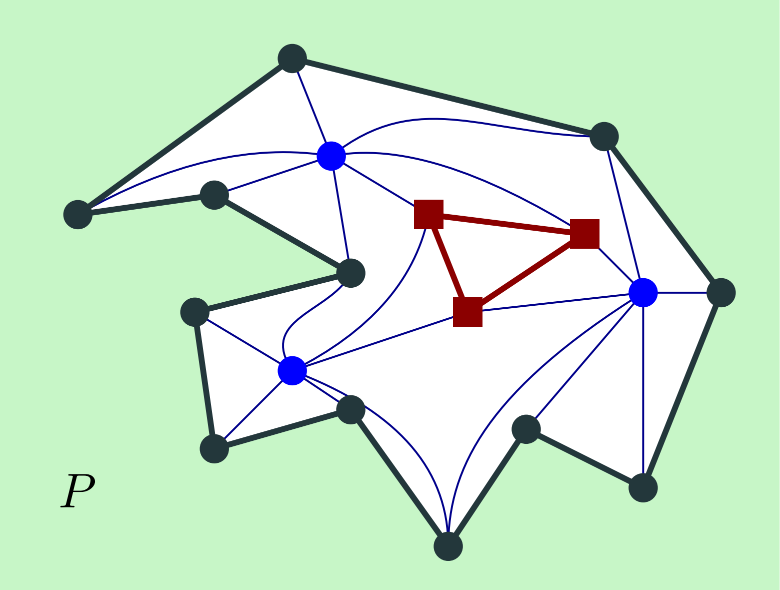
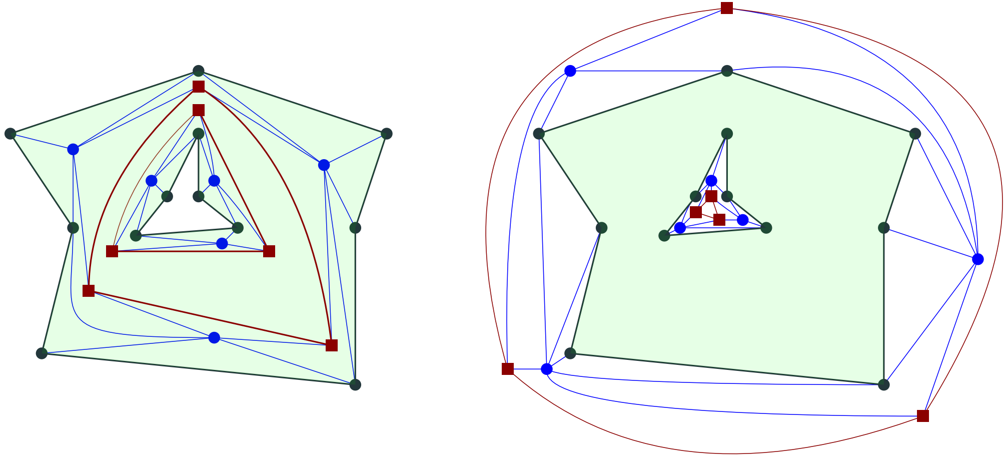


Inside and outside scaffolding



Application to problems

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- $(1+\varepsilon)$**-approx SSSP** - replace with ε-emulator of Chang et al. [CKT22].

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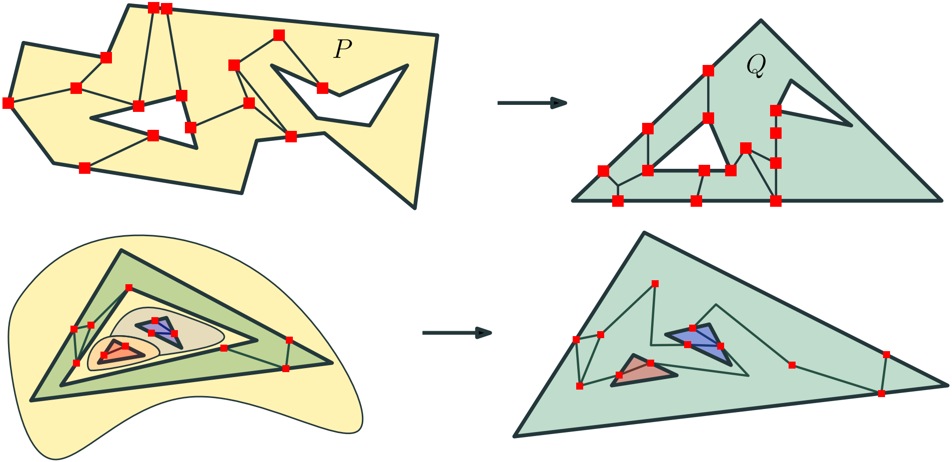
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