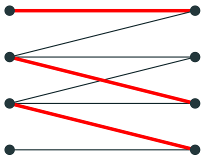
Multiplicative Auction Algorithms

Approximate Maximum Weight Bipartite Matching



Da Wei Zheng (UIUC) and Monika Henzinger (ISTA)

Sep 13, 2023

Paper presented at IPCO 2023

Matchings in bipartite graphs

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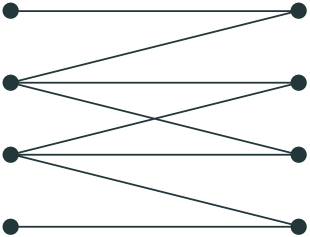
with $n = |U \cup V|$, $m = |E|$.



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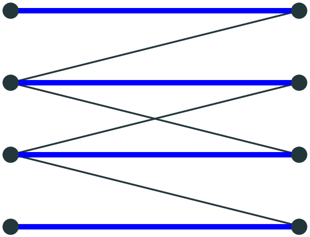

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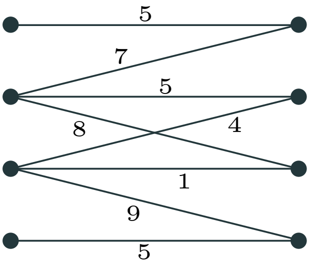

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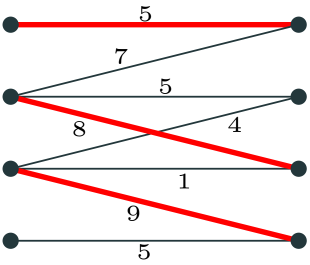

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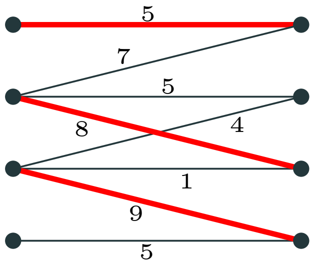

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Maximum Weight Matching (MWM)

Today: $(1 - \varepsilon)$-approximate maximum weight matching

Goal: Find a matching M such that: $$w(M) \geq (1 - \varepsilon)w(M^*)$$

History of Exact Bipartite MWM Algorithms

Year	Authors	Time bound
1890	Jacobi (written ∼1836)	$\mathrm{poly}(n)$
1946	Easterfield	$2^n \mathrm{poly}(n)$
1953-64	von Neumann, Kuhn, Gleyzal, Munkres, Balinsky–Gomory	$\mathrm{poly}(n)$
1969	Dinic–Kronrod	$O(n^3)$
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1983	Gabow	$O(mn^{3/4} \log W)$
1988-97	Gabow–Tarjan, Orlin–Ahuja, Goldberg–Kennedy	$O(m\sqrt{n} \log(nW))$
1996	Cheriyan–Melhorn	$\widetilde{O}(n^{5/2} \log(nW))$
2006	Kao–Lam–Sung–Ting, Sankowski	$O(n^\omega W)$
2012	Duan–Su	$O(m\sqrt{n} \log W)$

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2020	vd Brand-Lee-Nanogkai-Peng-Saranurak-Sidford-Song-Wang	$\widetilde{O}(m+n^{1.5})$
2022	Chen–Kyng–Liu–Peng–Probst Gutenberg–Sachdeva	$m^{1+o(1)}$

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2004	Pettie–Sanders	$2/3-\varepsilon$	$O(m \log \varepsilon^{-1})$
2010	Duan–Pettie, Hange–Hougardy	$3/4-\varepsilon$	$O(m \log n \log \varepsilon^{-1})$
2014	Duan–Pettie	$1-\varepsilon$	$O(m\varepsilon^{-1} \log \varepsilon^{-1})$

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2023	This talk (Bipartite only)	$1-\varepsilon$	$O(m\varepsilon^{-1})$

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Results

[Wajc '20], [ACCSW '18], [BhaK '21], [PelS '16] [AAGPS '19], [BeFH '19], [ChaS '18], [NeiS '16], [Sank '16], [BhHN '16], [BaGS '11], [BhHN '17], [BhaK '19], [BDHSS '19], [Solo '16], [BhCH '17], [BerS '15], [BerS '16], [Kiss '22], [GLSSS '19], [BehK '22], [BeLM '22], [RoSW '22], [BeRR '22], [GupP '13], ... and many more ...

Our results

1. A simple auction algorithm for $(1-\varepsilon)$-approximate MWM.
2. Efficient dynamic algorithm, supporting one-sided vertex deletion, and other-sided vertex insertion (simultaneously).

Multiplicative Auction Algorithm

The auction algorithm of Bertsekas ’81 and Demange–Gale–Sotomayor ’86

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While $\exists v \in V$ unallocated, $\textbf{util}(uv) > 0$, v bids $y_u + \delta$ and allocated max util u.

Left: Items $u \in U$

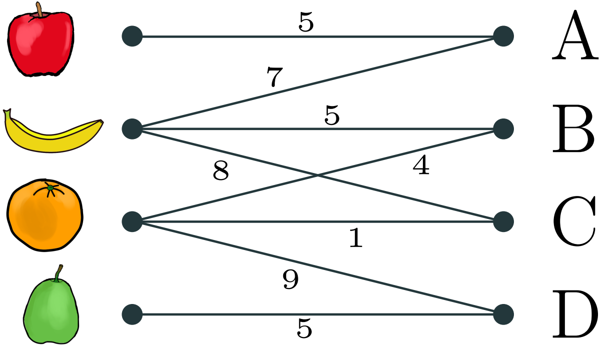
Price y_u initially 0

Utility of v having u:

$\textbf{util}(uv) = w(uv) - y_u$

Right: Bidders $v \in V$

Initially unallocated



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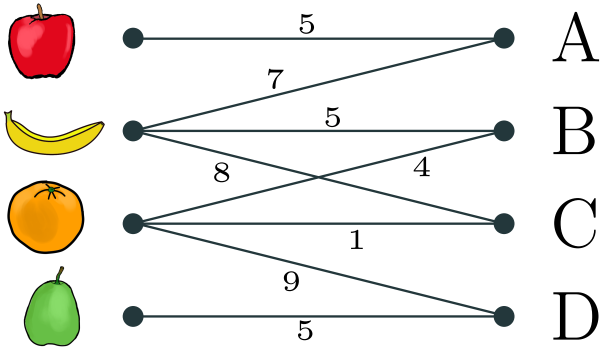

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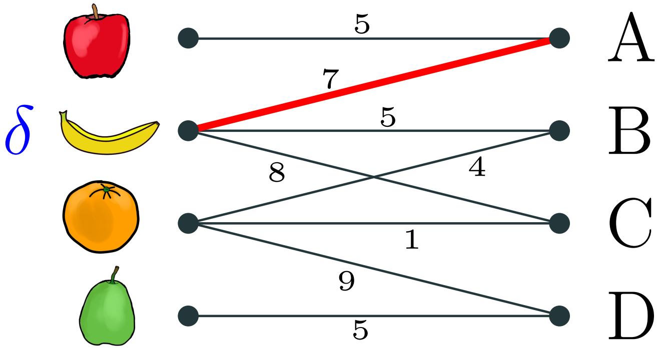

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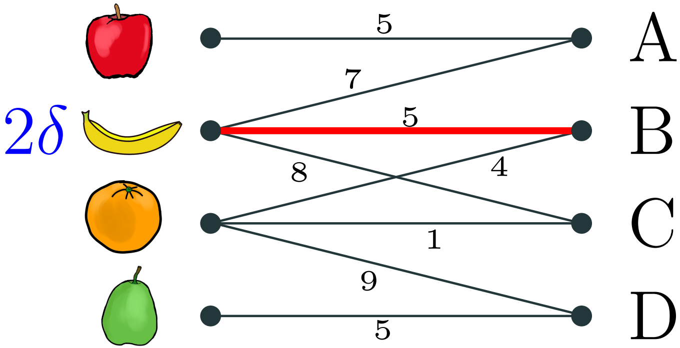

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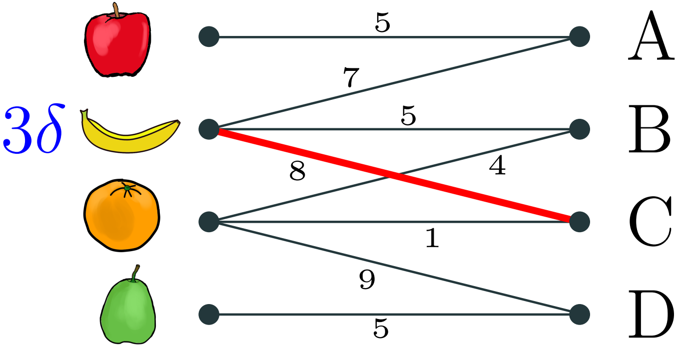

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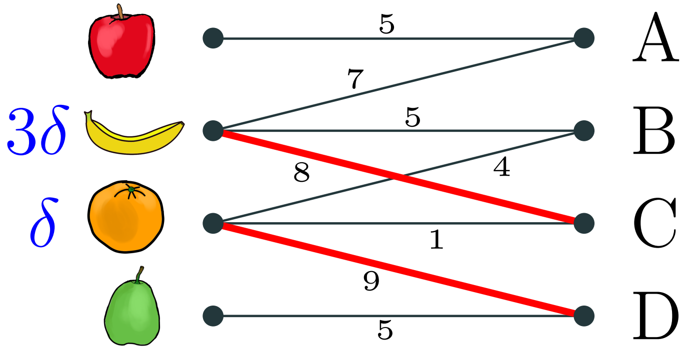

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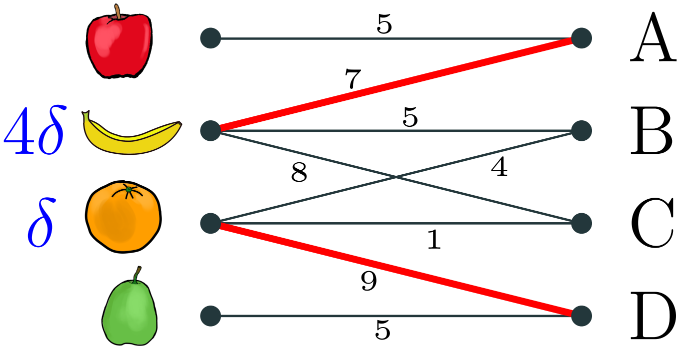

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Can be implemented in time $O(m\varepsilon^{-1})$, gets multiplicative error of $(1 - \varepsilon)$.

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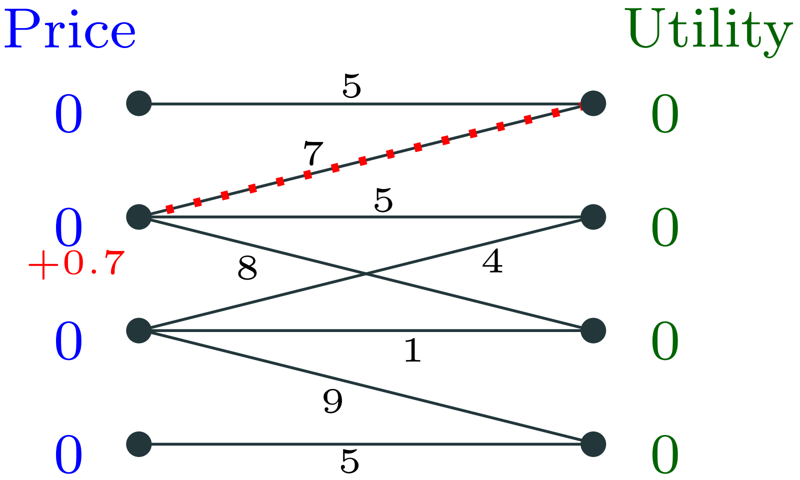

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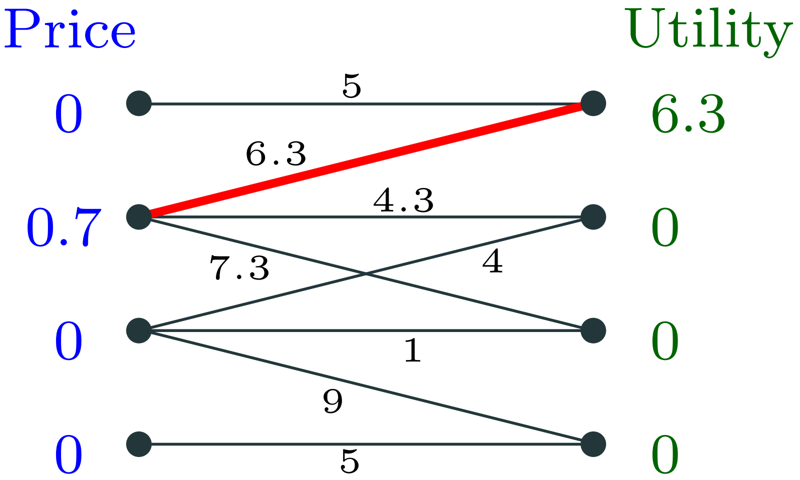

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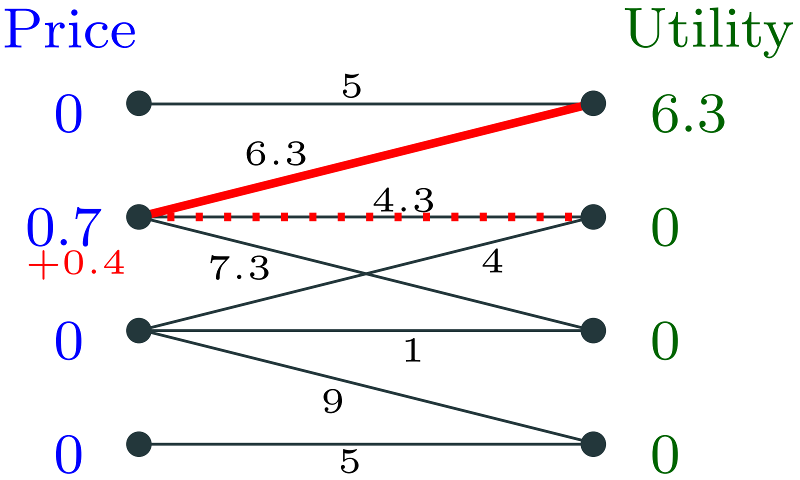

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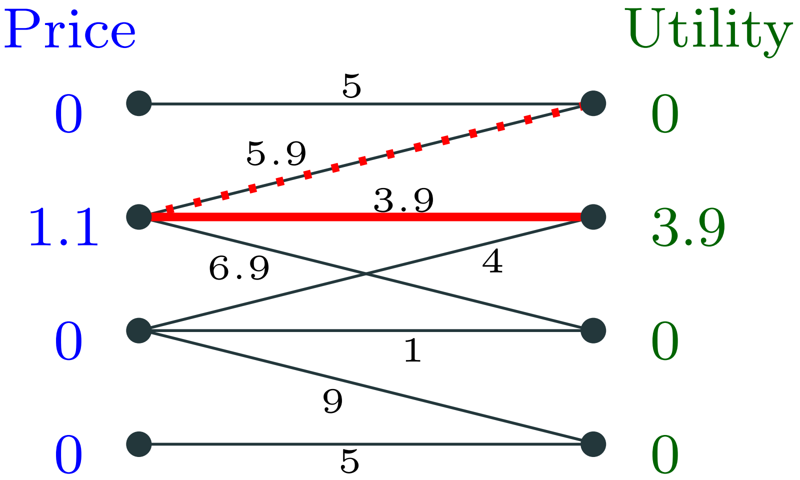

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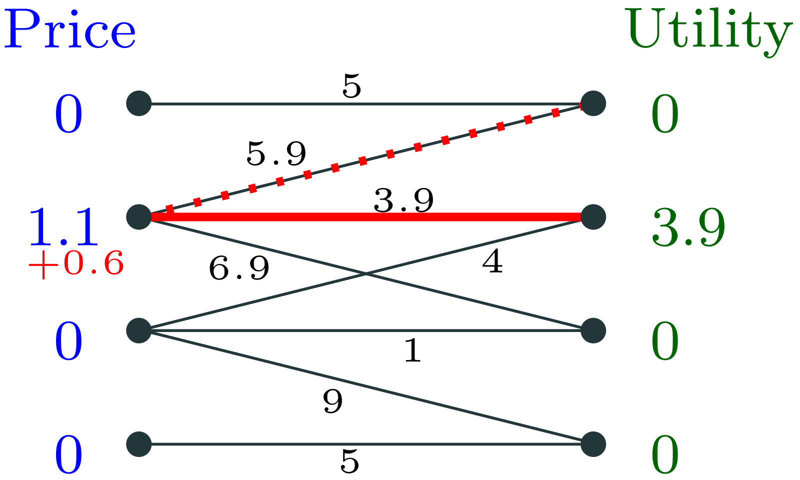

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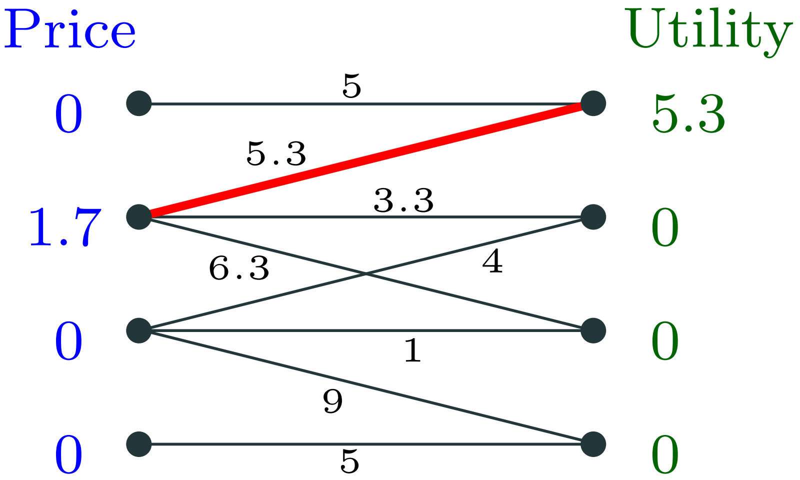

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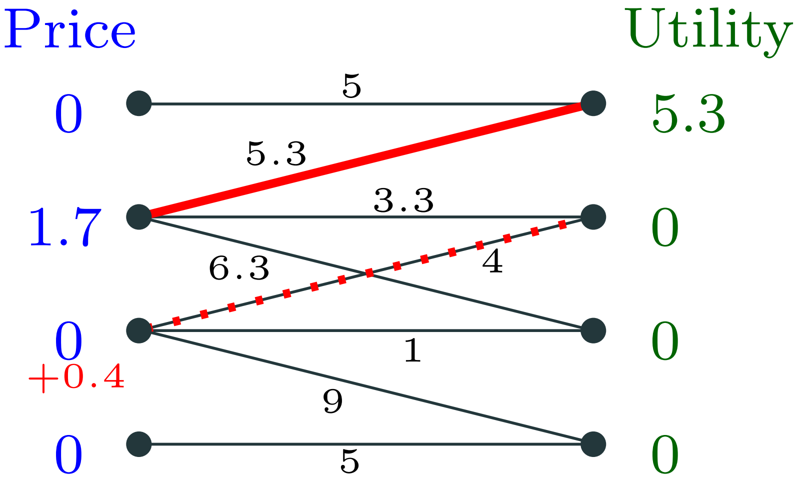

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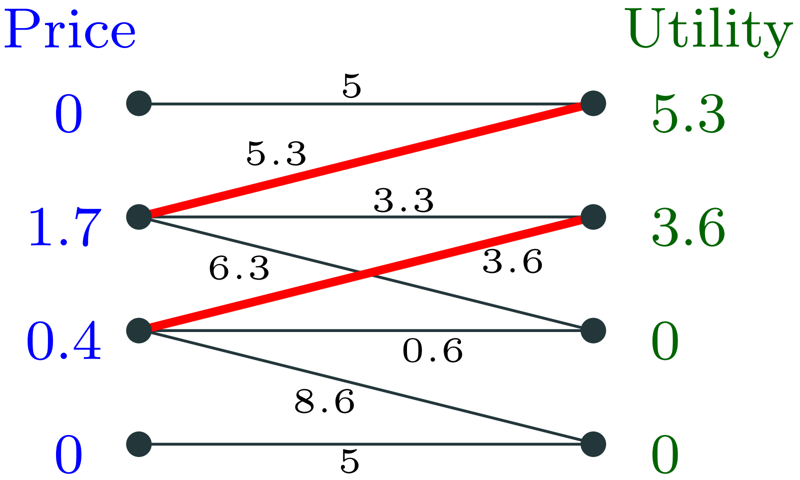

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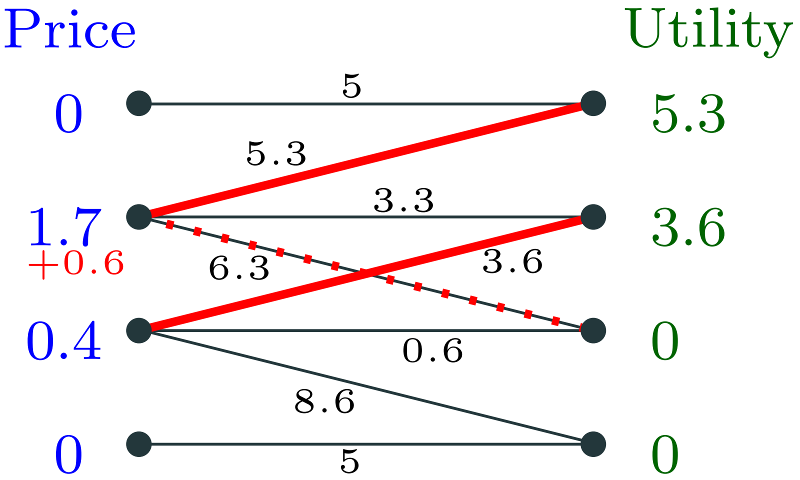

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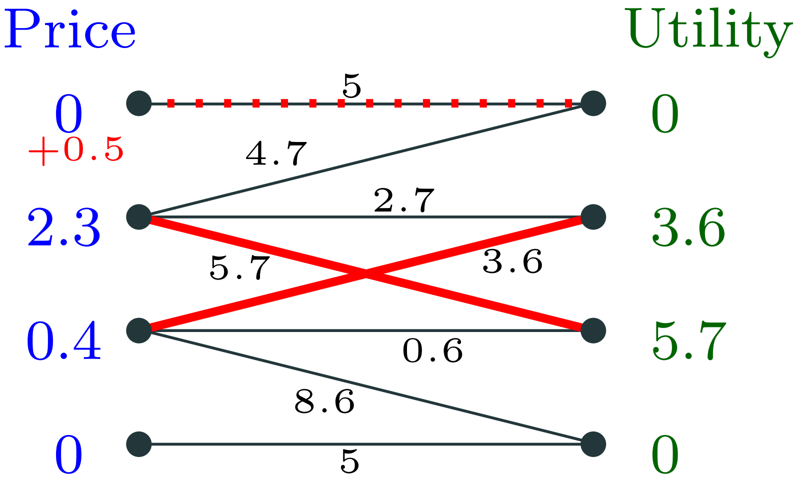

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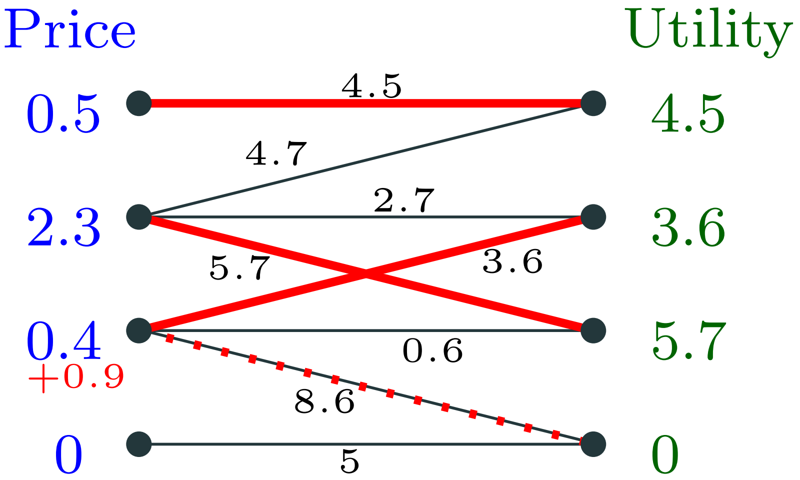

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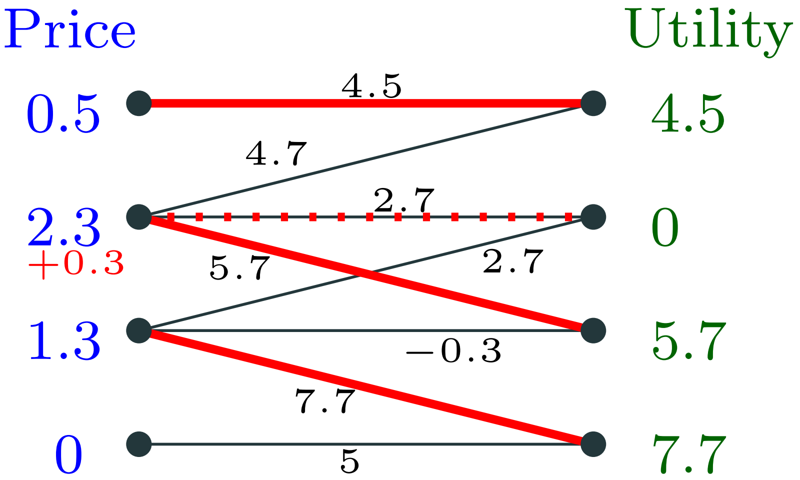

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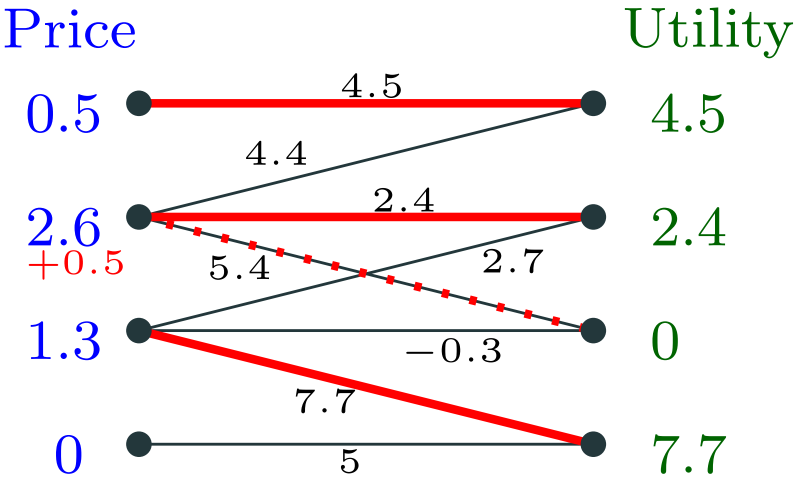

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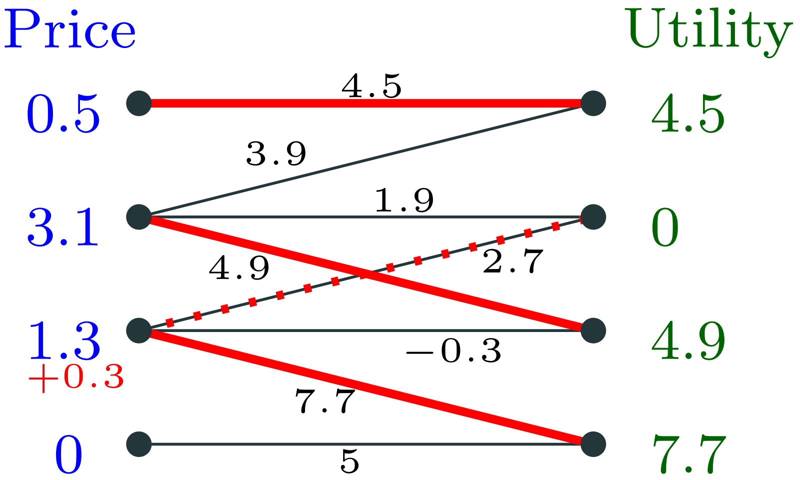

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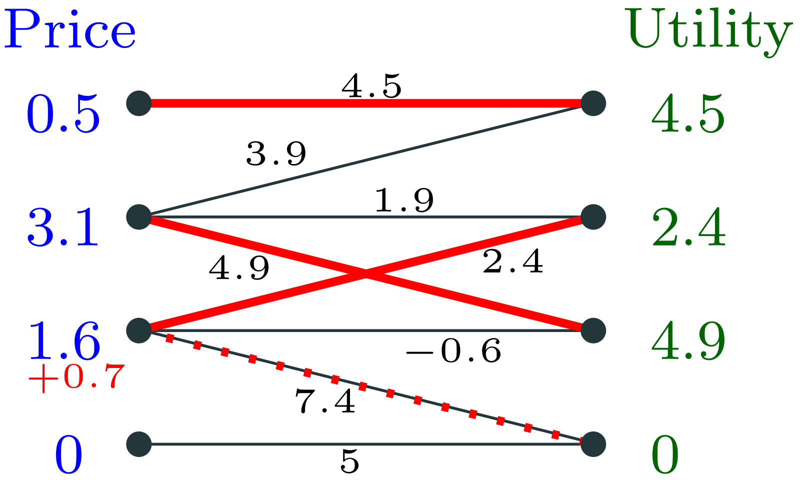

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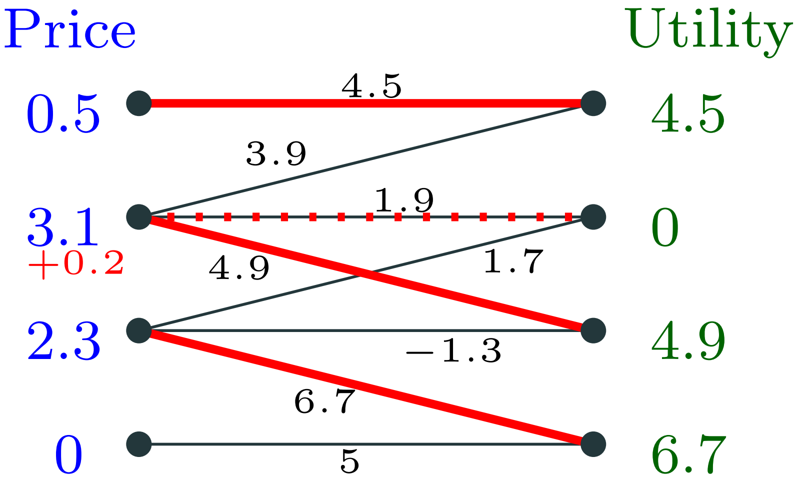

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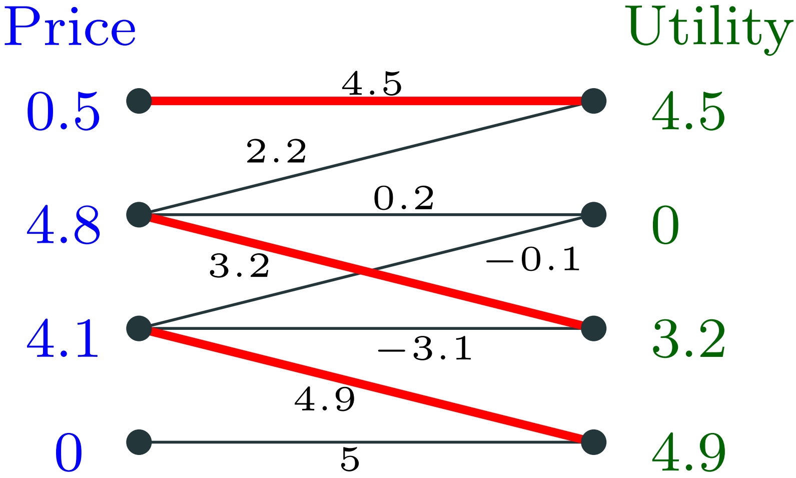

Implementation and runtime of the algorithm

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   We can also round these to powers of $(1+\varepsilon)$.

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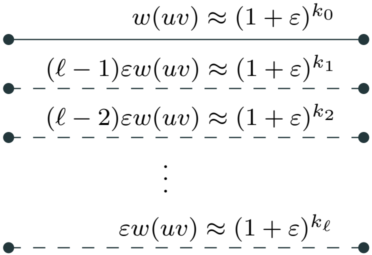

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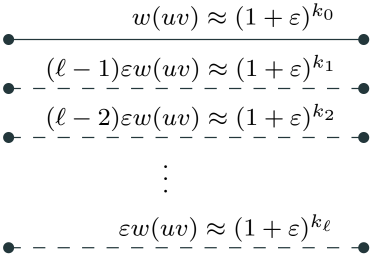

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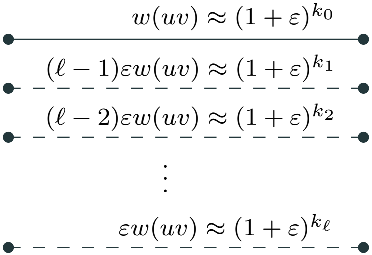

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$O(m\varepsilon^{-1})$ to sort integers in $[0, \varepsilon^{-1} \log n]$, and $O(m\varepsilon^{-1})$ for the algorithm.

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Adding a new vertex $v \in V$ along with incident edges

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Correctness of the algorithm

LP for MWM

Variables x_{uv} for each edge $uv \in E$.

$$
\begin{aligned}
\max \quad & \sum_{uv \in E} w(uv) x_{uv} & \\
s.t. \quad & \sum_{v \in N(u)} x_{uv} \leq 1 & \forall u \in U \\
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Variables y_u for $u \in U$, y_v for $v \in V$.

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\min \quad & \sum_{u \in U} y_u + \sum_{v \in V} y_v & \\
s.t. \quad & y_u + y_v \geq w(uv) & \forall uv \in E \\
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Let M^* be the maximum weight matching.

$$\begin{aligned}
w(M) &= \sum_{uv \in M} w(uv) \\
&\geq \sum_{uv \in M} (1+\varepsilon_0)^{-1} \cdot (y_u + y_v) && \text{Approx. comp. slackness} \\
&= (1+\varepsilon_0)^{-1} \sum_{z \in U \cup V} y_z && \text{Complementarity} \\
&\geq (1+\varepsilon_0)^{-1} \sum_{uv \in M^*} (y_u + y_v) && \text{Non-negativity of } y_z \\
&\geq (1+\varepsilon_0)^{-1} (1-\varepsilon_1) \cdot w(M^*) && \text{Approximate dominance}
\end{aligned}$$

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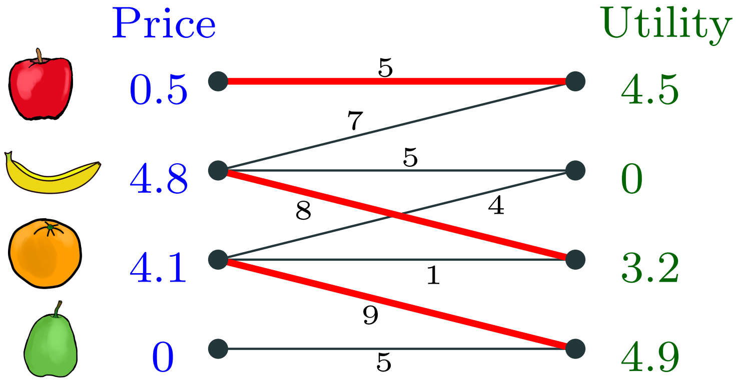

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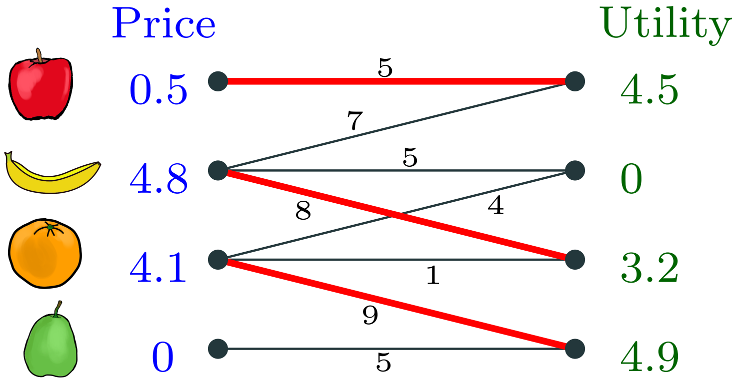

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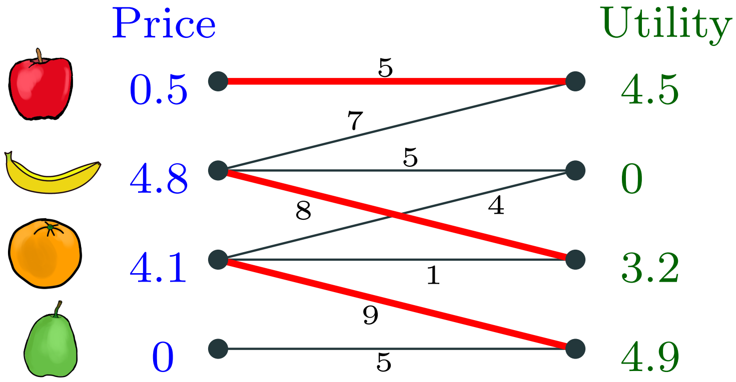
uv is in the matching.

$$y_u + \textbf{util}(uv) = w(uv).$$



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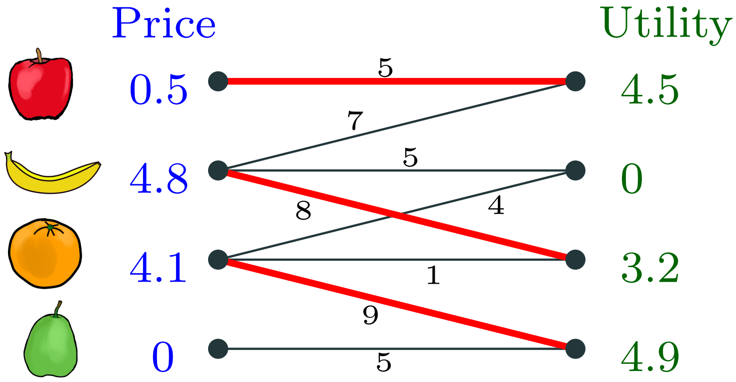

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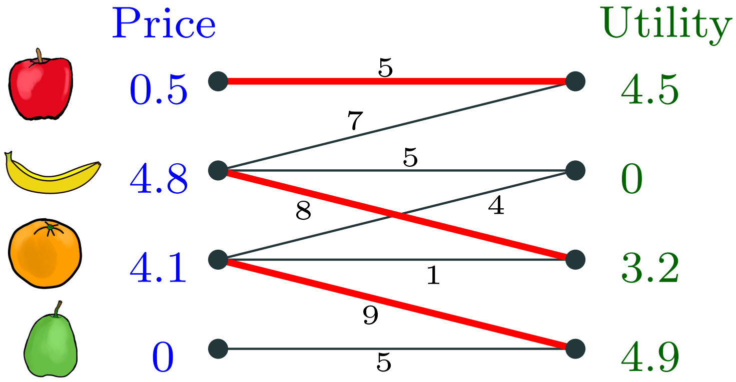
All items have high price.



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5. (Decremental / incremental) $(1-\varepsilon)$-approximate SSSP / transshipment?