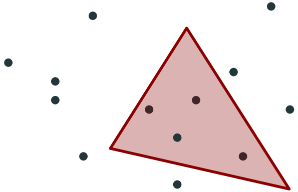
Simplex Range Searching Revisited



Shaving Logs in Multi-Level Data Structures

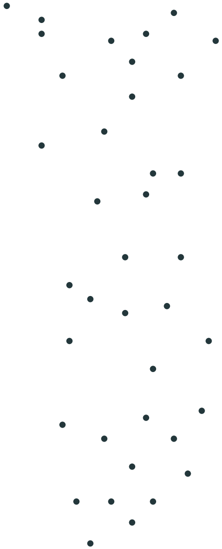
Timothy M. Chan and **Da Wei Zheng**

January 22, 2023

University of Illinois Urbana-Champaign

Range Searching

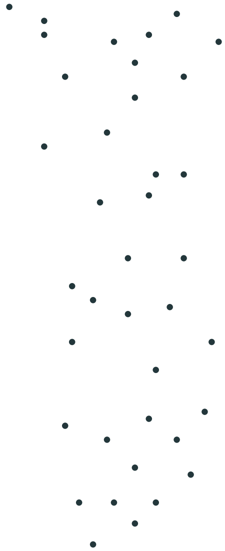
Input: n points in $\mathbb{R}^2$ or $\mathbb{R}^d$



Range Searching

Input: n points in $\mathbb{R}^2$ or $\mathbb{R}^d$

Given a range which could be:



Range Searching

Input: n points in $\mathbb{R}^2$ or $\mathbb{R}^d$

Given a range which could be:

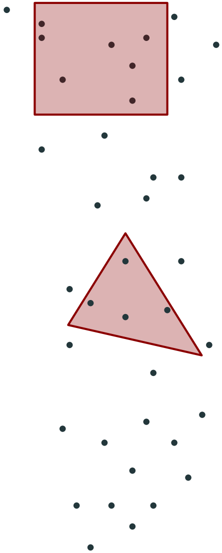
- Orthogonal

Range Searching

Input: n points in $\mathbb{R}^2$ or $\mathbb{R}^d$

Given a range which could be:

- Orthogonal
- Simplex



Range Searching

Input: n points in $\mathbb{R}^2$ or $\mathbb{R}^d$

Given a range which could be:

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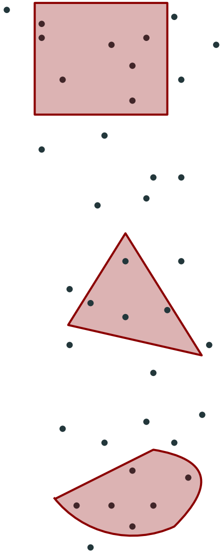
- Simplex
- Semi-algebraic



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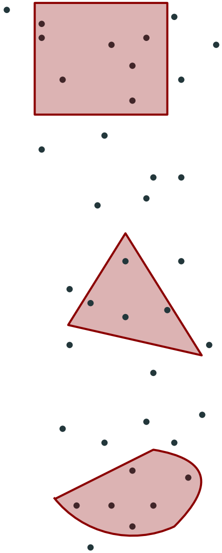
Answer queries of the form:

- Counting - Output the number of points in the range.



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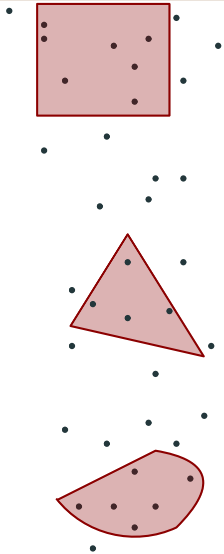
Answer queries of the form:

- Counting - Output the number of points in the range.
- Reporting - Report all points in the range.



Range Searching

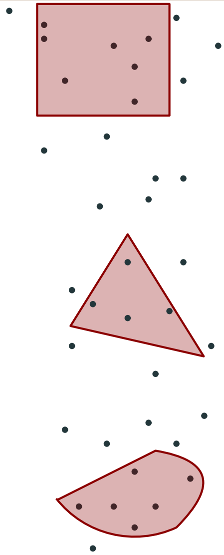
Input: n points in $\mathbb{R}^2$ or $\mathbb{R}^d$

Given a range which could be:

- Orthogonal
- Simplex
- Semi-algebraic

Answer queries of the form:

- Counting - Output the number of points in the range.
- Reporting - Report all points in the range.
- Semigroup - Each point has value in (commutative) semigroup, report result after applying semigroup operation to all points within range.



History of Simplex Range Searching (Linear space)

Problem	Space	Time	Reference
$\mathbb{R}^2$	n	$n^{0.793}$	[Willard '82]

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Problem	Space	Time	Reference
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	n	$n^{1-1/d}(\log n)^c$	[Matoušek '92]

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	n	$n^{1-1/d}$	[Matoušek '93]

History of simplex range searching (Fast query time)

Problem	Space	Time	Reference
$\mathbb{R}^2$	$n^{7+\varepsilon}$	$\log n$	[Edelsbrunner Maurer Kirkpatrick '82]

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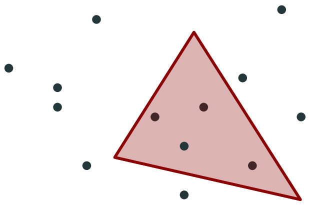
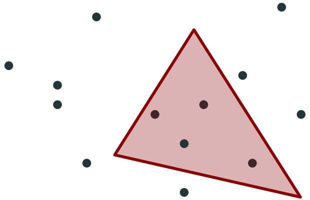
	n^d	$\log^{d+1} n$	[Matoušek '93]

Simplex Range Searching

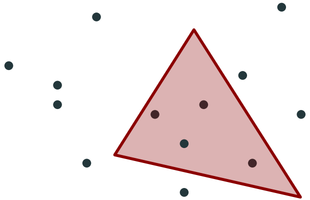


Simplex Range Searching



Problem	Space	Time	Reference
Counting	$n^{d+\varepsilon}$	$\log n$	[CSW '92]
	n^{d}	$\log^{d+1} n$	[Matoušek'93]

Simplex Range Searching



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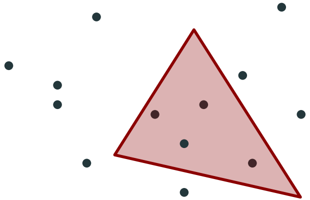

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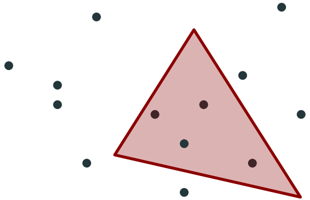
Is there a $O(n^d)$ space data structure with $O(\log n)$ query time?

Simplex Range Searching



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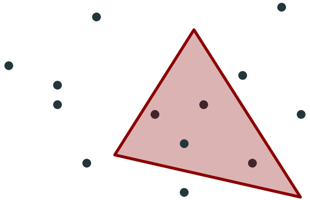
Counting	$n^{d+\varepsilon}$	$\log n$	[CSW '92]
	n^d	$\log^{d+1} n$	[Matoušek'93]
	n^d	$\log n$	new

Is there a $O(n^d)$ space data structure with $O(\log n)$ query time?

YES!

Simplex Range Searching



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Counting	$n^{d+\varepsilon}$	$\log n$	[CSW '92]
	n^d	$\log^{d+1} n$	[Matoušek'93]
	n^d	$\log n$	new
Reporting	$n^{d+\varepsilon}$	$\log n + k$	[CSW '92]
	n^d	$\log^{d+1} n + k$	[Matoušek'93]
	n^d	$\log n + k$	new

Is there a $O(n^d)$ space data structure with $O(\log n)$ query time?

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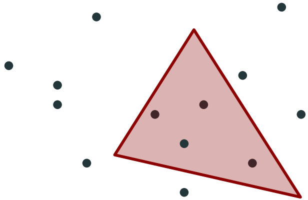

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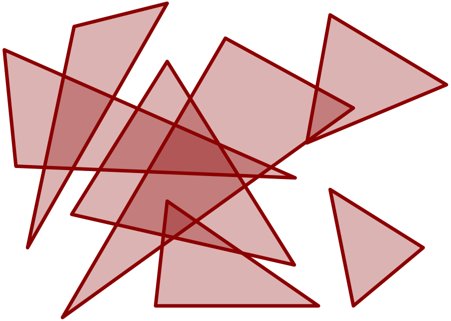
	n^d	$\log n + k$	new
Semigroup	$n^{d+\varepsilon}$	$\log n$	[CSW '92]
	n^d	$\log^{d+1} n$	[Matoušek'93]
	$\widetilde{O}(n^d)$	$\log n$	new

Is there a $O(n^d)$ space data structure with $O(\log n)$ query time?

YES!

Simplex Stabbing



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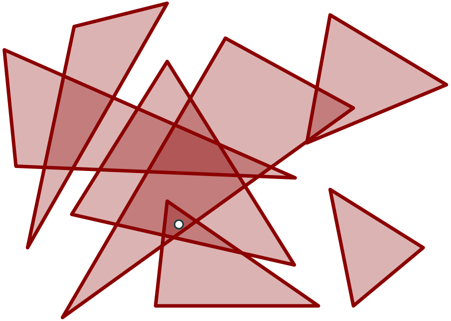

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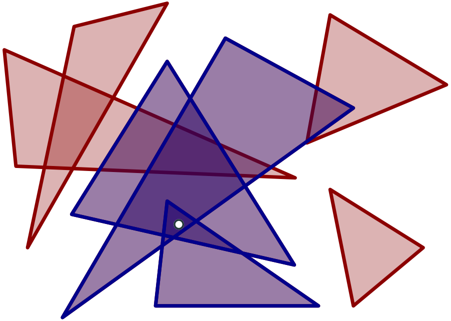
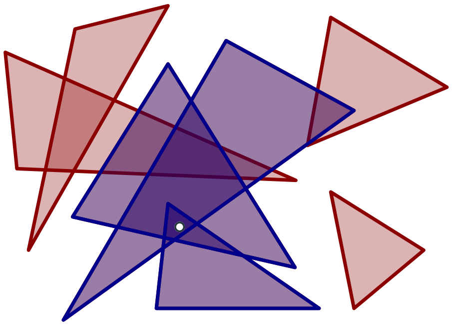


Simplex Stabbing

reporting $O(n \log^d n)$ space $O(n^{1-1/d} \log^d n + k)$ time [Chan '12]



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reporting	$O(n \log^d n)$	space	$O(n^{1-1/d} \log^d n + k)$	time	[Chan '12]
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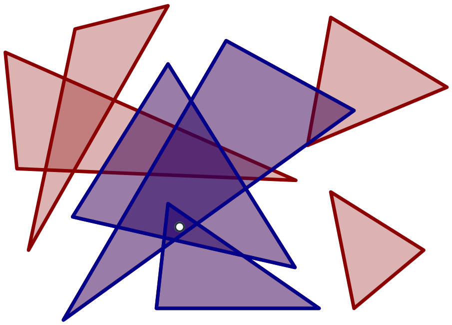
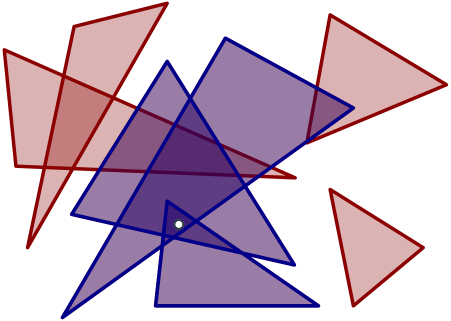


Simplex Stabbing

reporting	$O(n \log^d n)$	space	$O(n^{1-1/d} \log^d n + k)$	time	[Chan '12]
	$O(n)$	space	$O(n^{1-1/d} + k)$	time	new
counting	$O(n \log^d n)$	space	$O(n^{1-1/d} \log^d n)$	time	[Chan '12]



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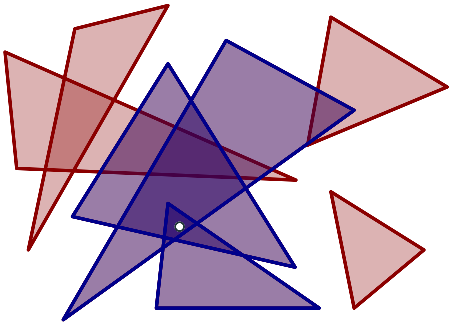
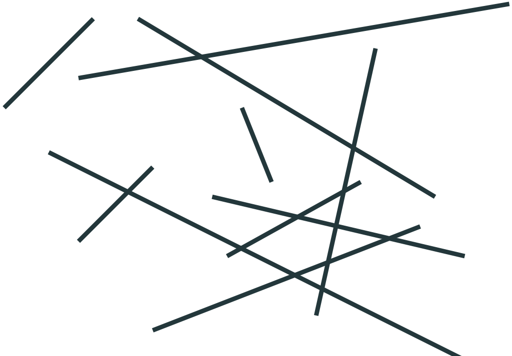
reporting	$O(n \log^d n)$	space	$O(n^{1-1/d} \log^d n + k)$	time	[Chan '12]
	$O(n)$	space	$O(n^{1-1/d} + k)$	time	new
counting	$O(n \log^d n)$	space	$O(n^{1-1/d} \log^d n)$	time	[Chan '12]
	$O(n)$	space	$O(n^{1-1/d})$	time	new



Line Segment Intersection



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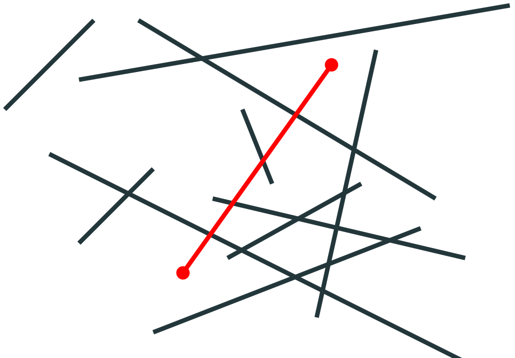

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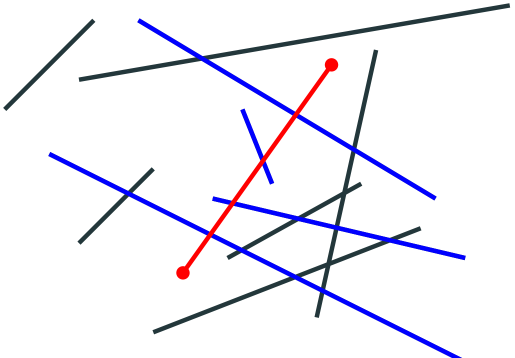
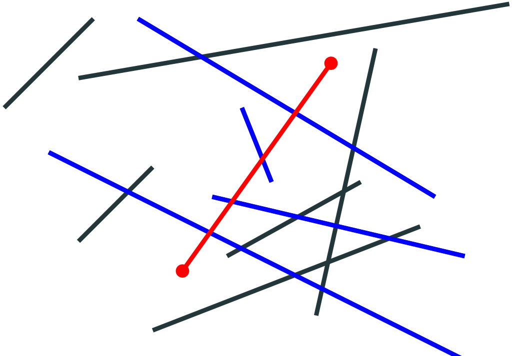


Line Segment Intersection

reporting $O(n \log^2 n)$ space $O(\sqrt{n} \log^2 n + k)$ time [Cheng Janardan '92]



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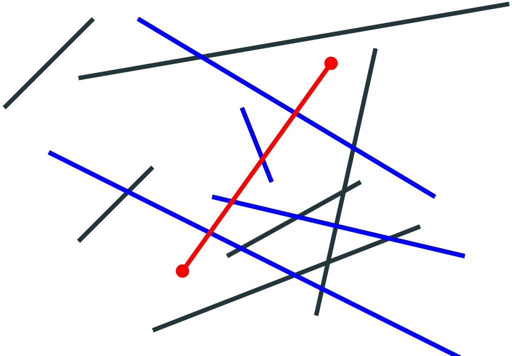
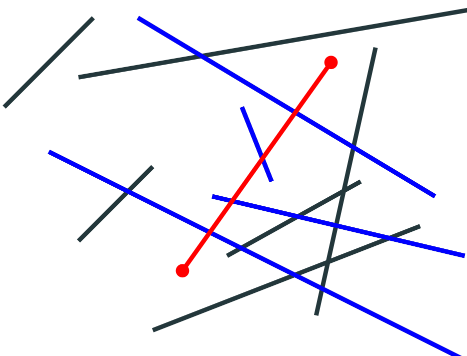


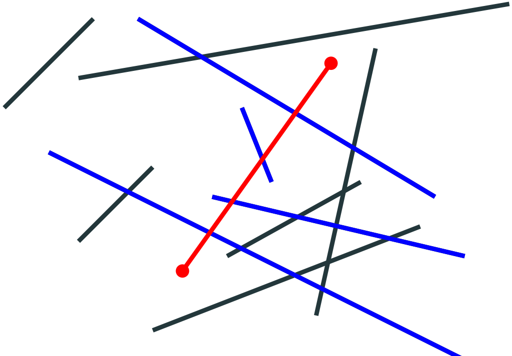
Line Segment Intersection

reporting	$O(n \log^2 n)$	space	$O(\sqrt{n} \log^2 n + k)$	time	[Cheng Janardan '92]
counting	$O(n \log^2 n)$	space	$O(\sqrt{n} \log n)$	time	[Bar-Yehuda Fogel '94]



Line Segment Intersection

reporting	$O(n \log^2 n)$	space	$O(\sqrt{n} \log^2 n + k)$	time	[Cheng Janardan '92]
	$O(n)$	space	$O(\sqrt{n} + k)$	time	new
counting	$O(n \log^2 n)$	space	$O(\sqrt{n} \log n)$	time	[Bar-Yehuda Fogel '94]



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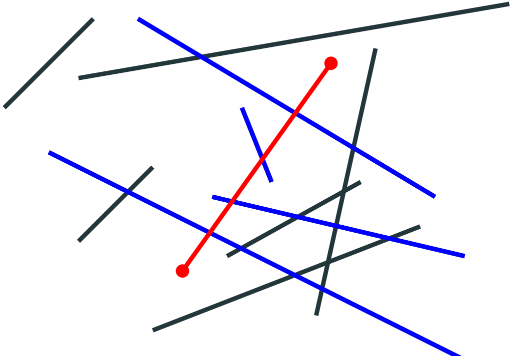
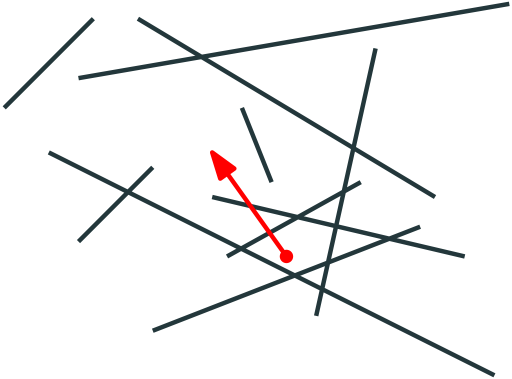
counting	$O(n \log^2 n)$	space	$O(\sqrt{n} \log n)$	time	[Bar-Yehuda Fogel '94]
	$O(n)$	space	$O(\sqrt{n})$	time	new



Ray Shooting



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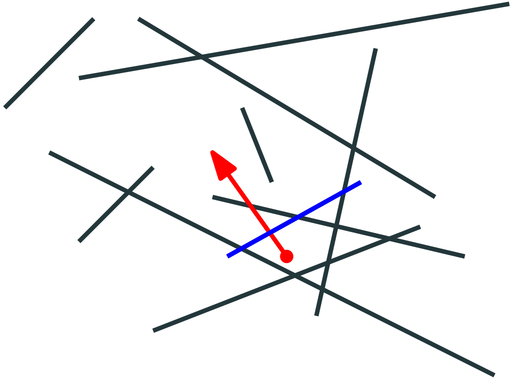
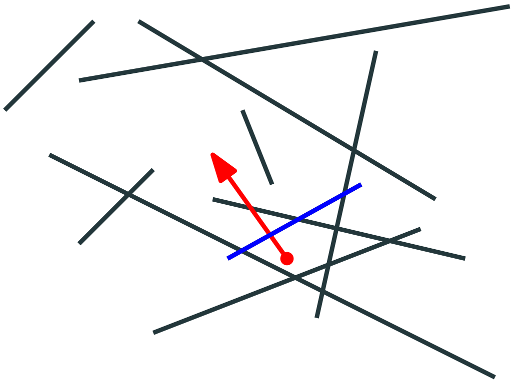


Ray Shooting

$O(n\,\alpha(n)\log^2 n)$ space $O(\sqrt{n\,\alpha(n)}\log n)$ time [Bar-Yehuda Fogel '94]



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$O(n\,\alpha(n) \log^2 n)$	space	$O(\sqrt{n\,\alpha(n)} \log n)$	time	[Bar-Yehuda Fogel '94]
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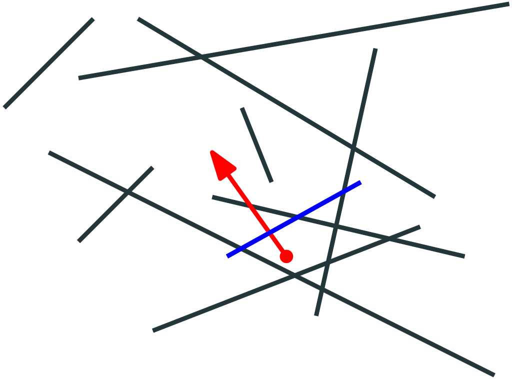

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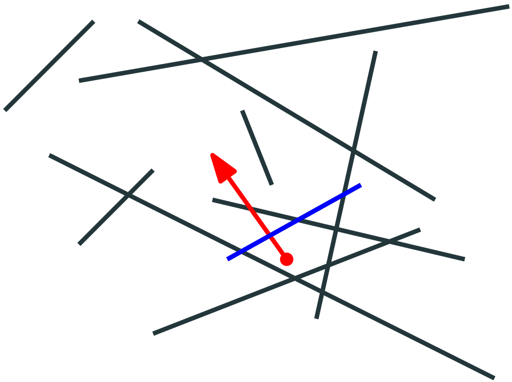
$O(n\log^2 n)$	space	$O(\sqrt{n}\log n)$	time	[Cheng Janardan '92]
$O(n\log n)$	space	$O(\sqrt{n}\log n)$	time	[Wang '20]



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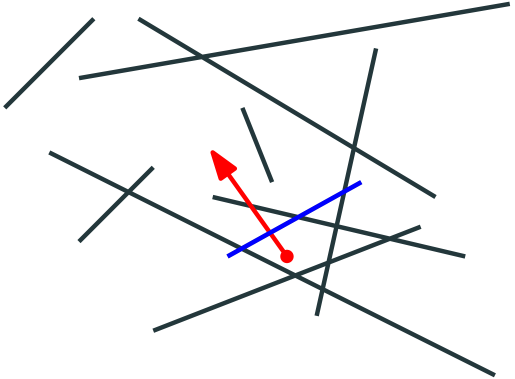
$O(n\log^2 n)$	space	$O(\sqrt{n}\log n)$	time	[Cheng Janardan '92]
$O(n\log n)$	space	$O(\sqrt{n}\log n)$	time	[Wang '20]
$O(n)$	space	$O(\sqrt{n})$	time	new



What do these problems have in common?

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They are solved with multi-level data structures!

What is a multi-level data structure?

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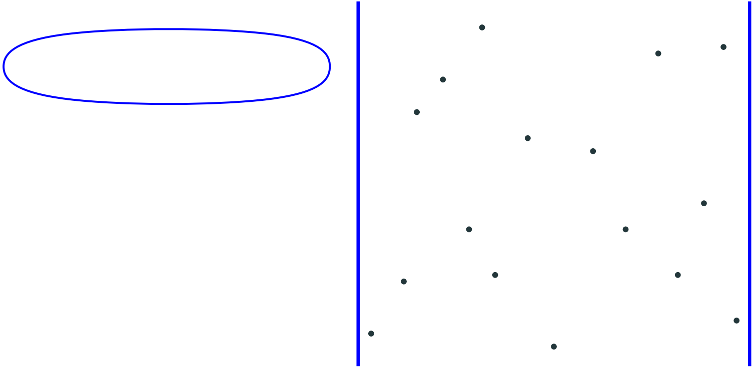
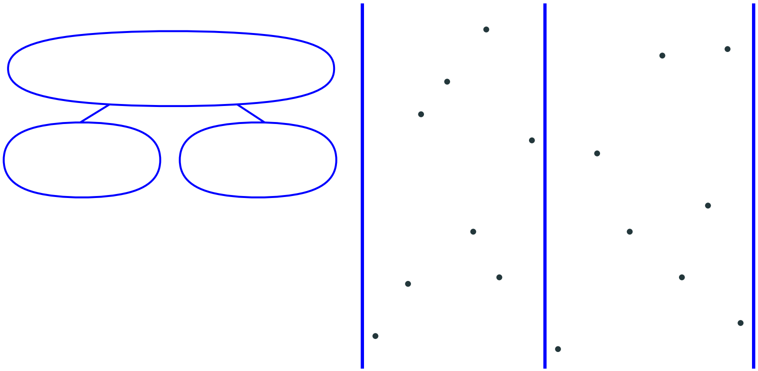


What is a multi-level data structure?



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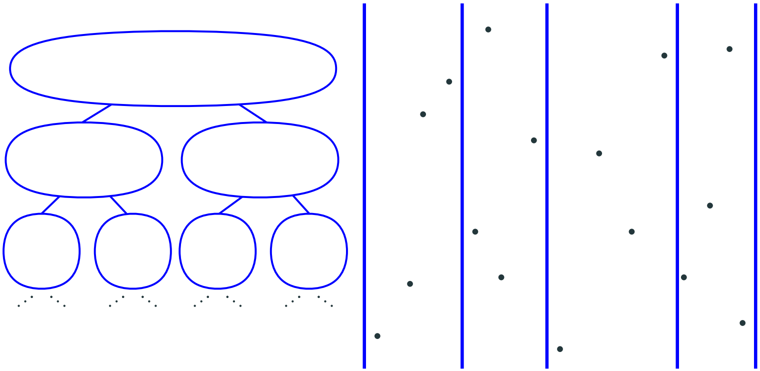

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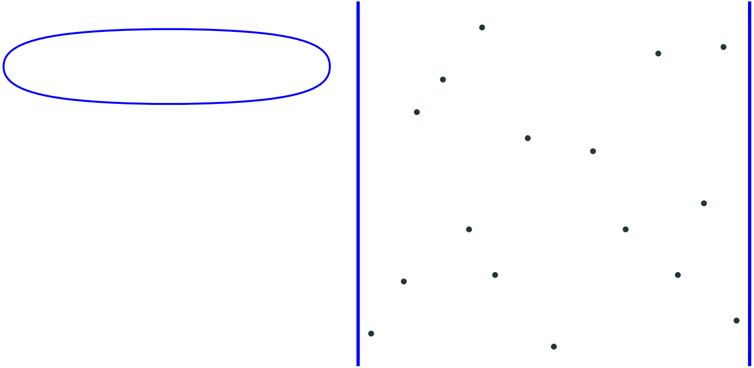

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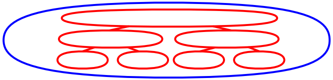

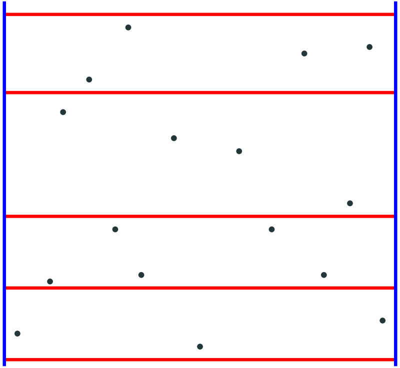

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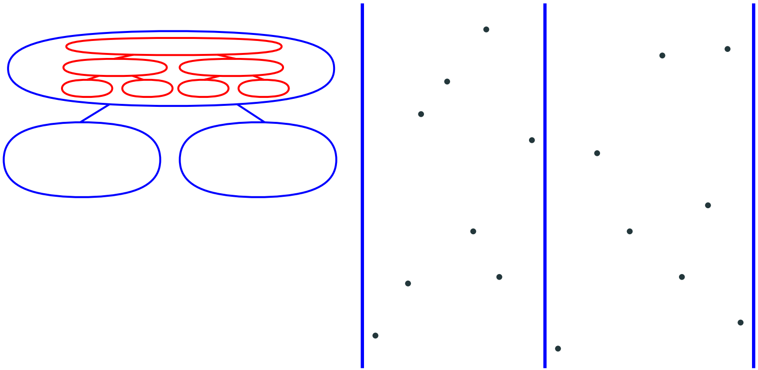

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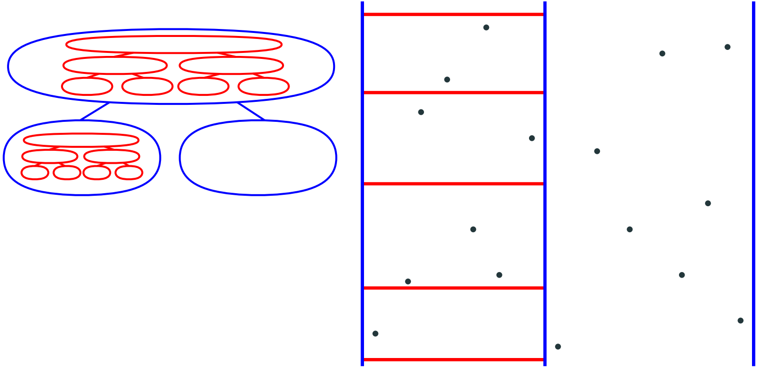

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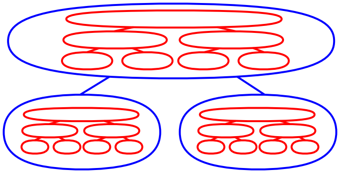

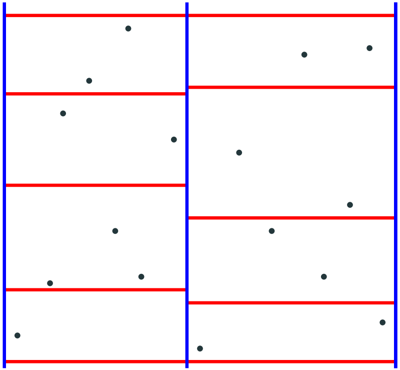

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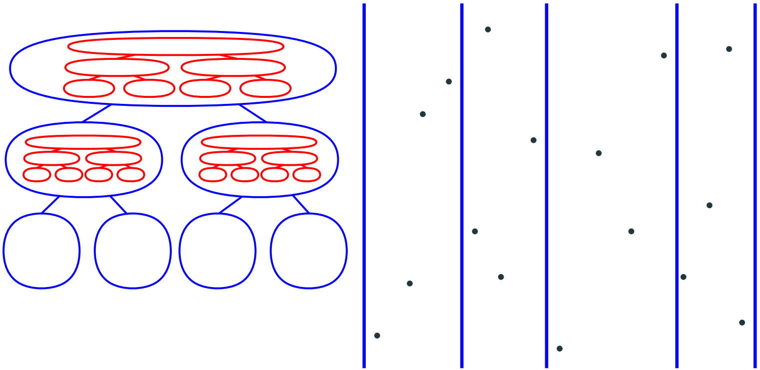

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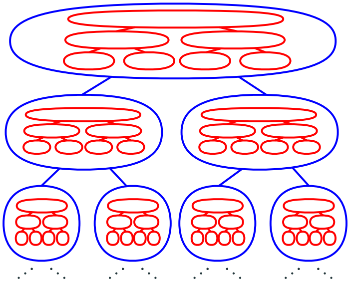

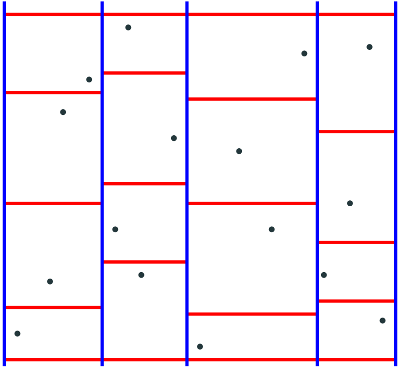

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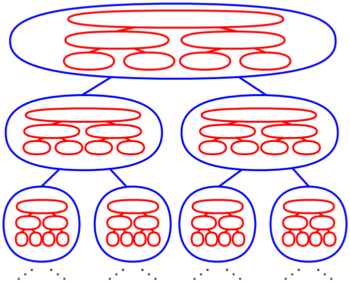

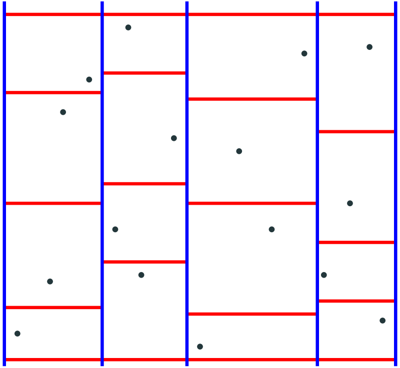

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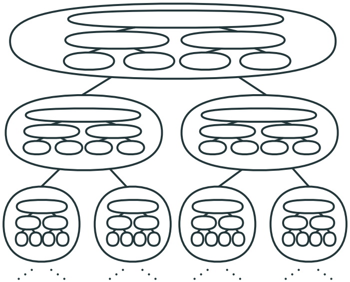

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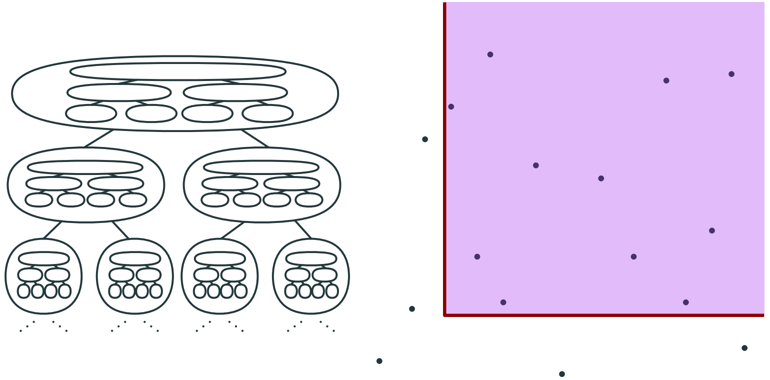

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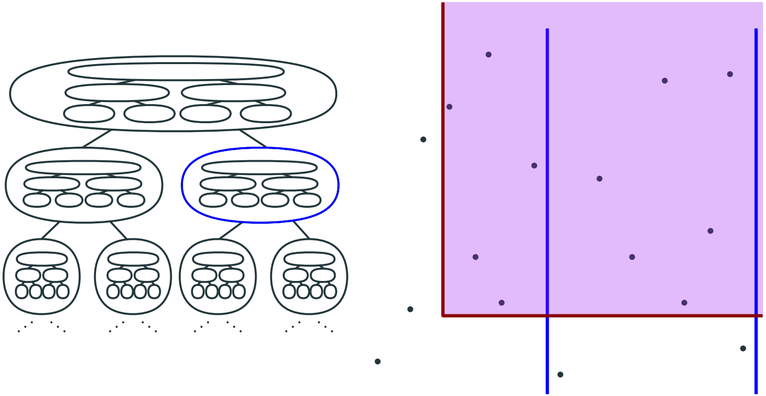

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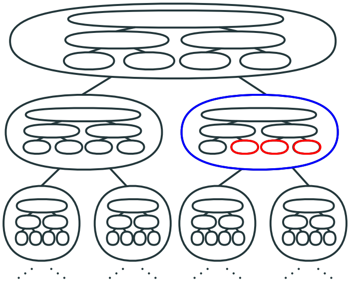

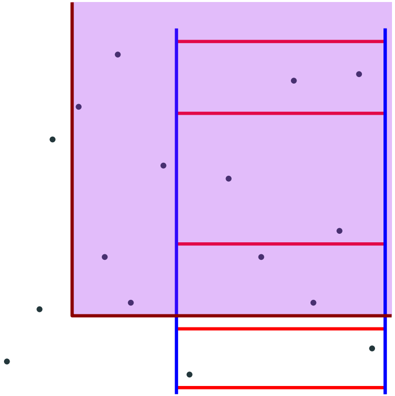

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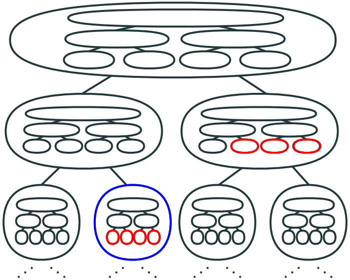

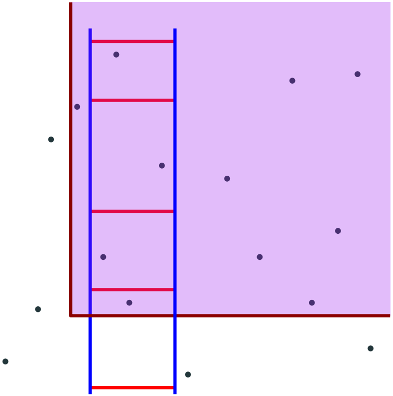

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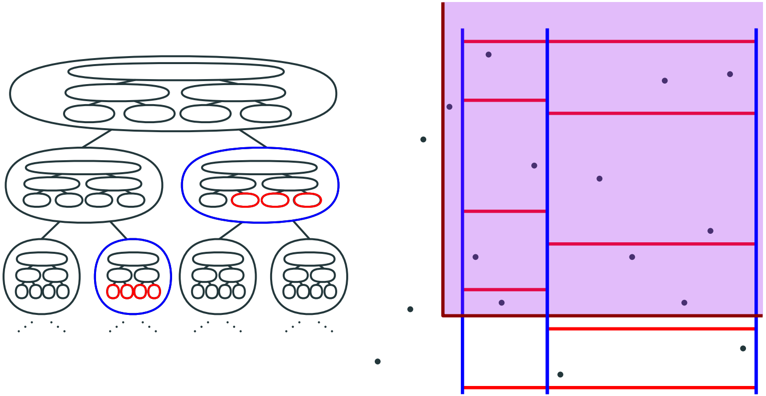

1D query data structure: $O(n)$ space $O(\log n)$ time / query

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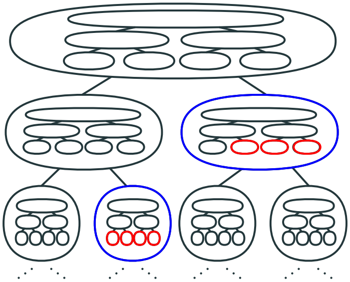
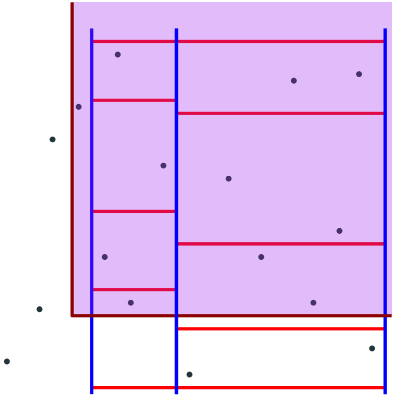
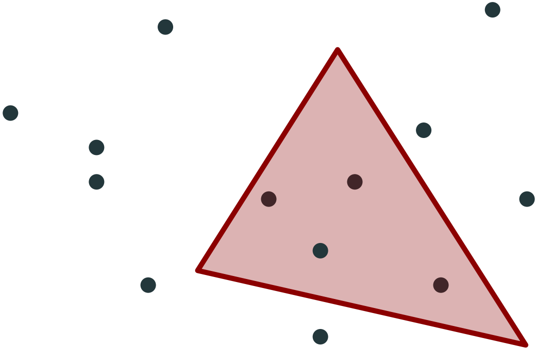




1D query data structure: $O(n)$ space $O(\log n)$ time / query

2D query data structure: $O(n \log n)$ space $O(\log^2 n)$ time / query

Simplex Range Query - Old method



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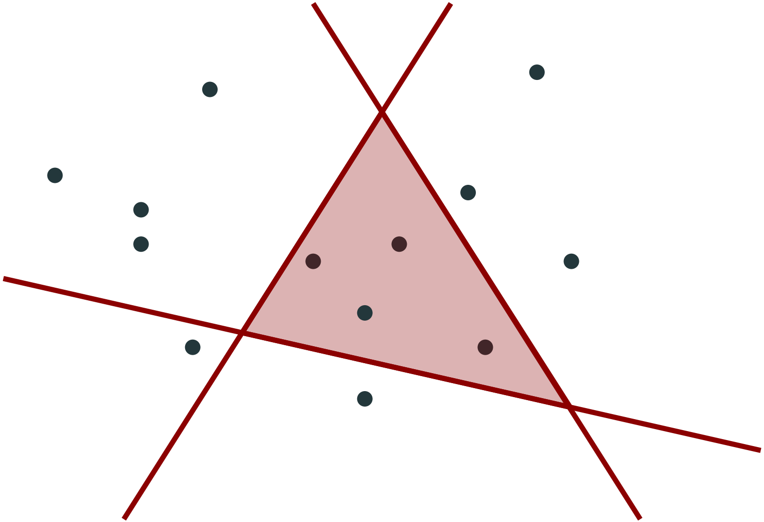

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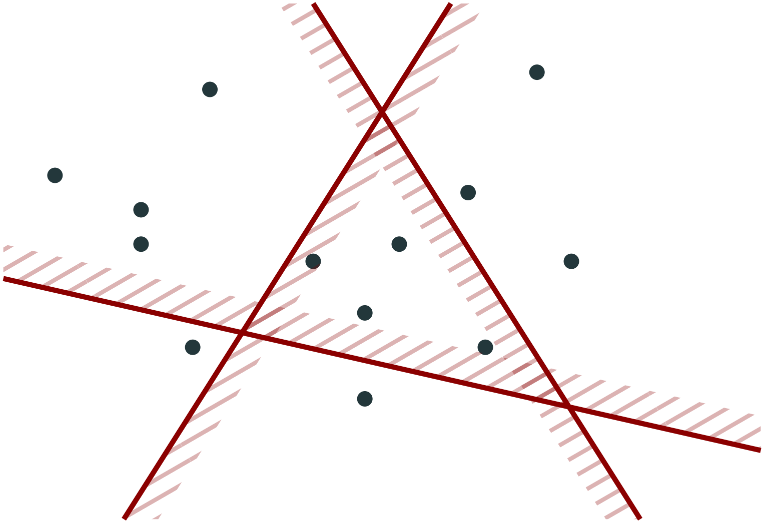

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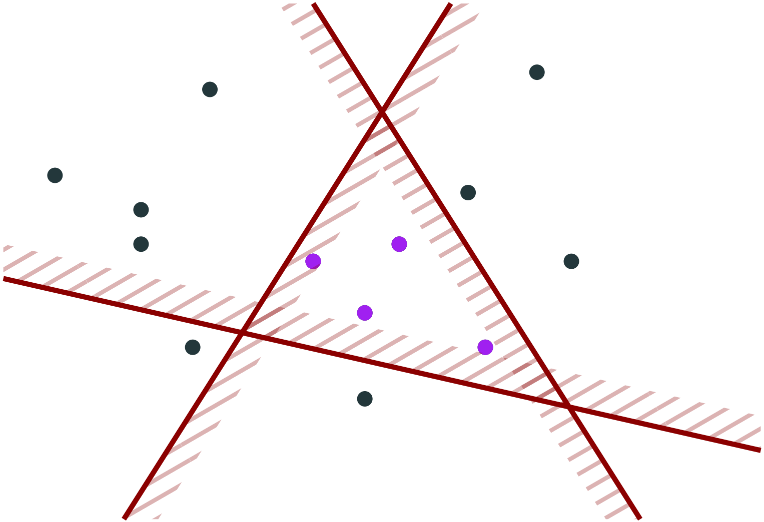

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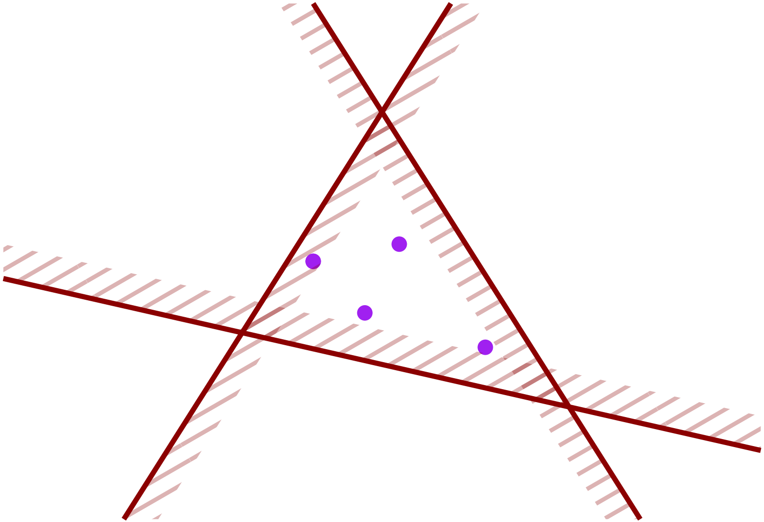
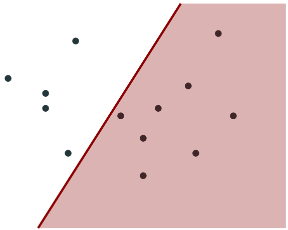


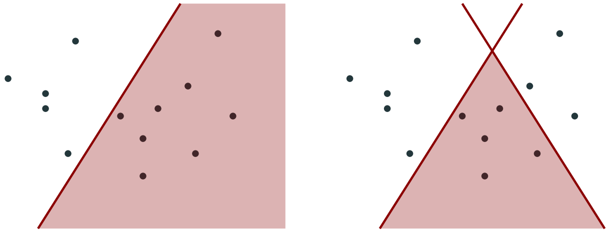
Simplex Range Query - Old method (cont.)

1-halfspace data structure $O(n^d)$ space $O(\log n)$ time / query



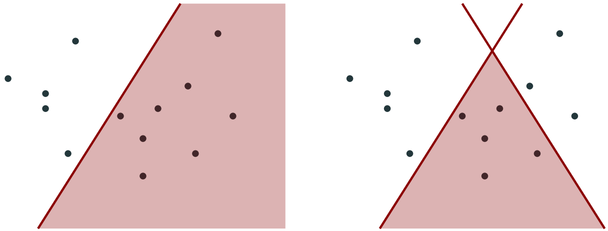
Simplex Range Query - Old method (cont.)

1-halfspace data structure	$O(n^d)$ space	$O(\log n)$ time / query
2-halfspace data structure	$O(n^d)$ space	$O(\log^2 n)$ time / query



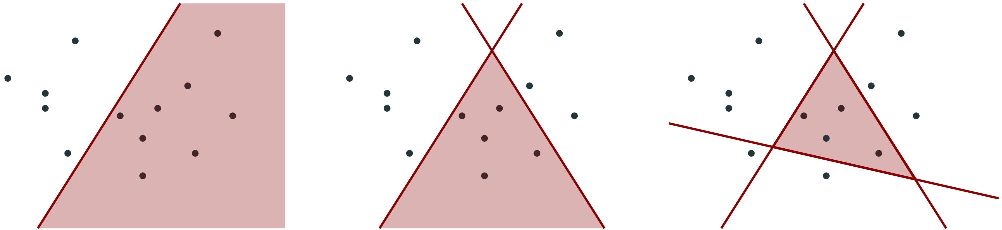
Simplex Range Query - Old method (cont.)

1-halfspace data structure	$O(n^d)$ space	$O(\log n)$ time / query
2-halfspace data structure	$O(n^d)$ space	$O(\log^2 n)$ time / query
...	...	...



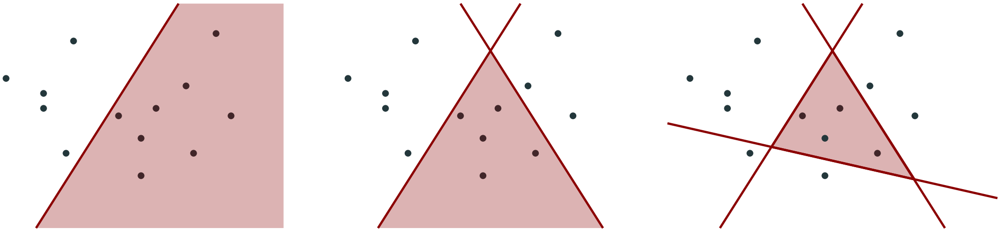
Simplex Range Query - Old method (cont.)

1-halfspace data structure	$O(n^d)$ space	$O(\log n)$ time / query
2-halfspace data structure	$O(n^d)$ space	$O(\log^2 n)$ time / query
...	...	...
d-halfspace data structure	$O(n^d)$ space	$O(\log^d n)$ time / query



Simplex Range Query - Old method (cont.)

1-halfspace data structure	$O(n^d)$ space	$O(\log n)$ time / query
2-halfspace data structure	$O(n^d)$ space	$O(\log^2 n)$ time / query
...	...	...
d-halfspace data structure	$O(n^d)$ space	$O(\log^d n)$ time / query
$(d+1)$-halfspace data structure	$O(n^d)$ space	$O(\log^{d+1} n)$ time / query



How did we improve this algorithm?

For multilevel data structures, if secondary structures have lower complexity, we don't lose log factors.

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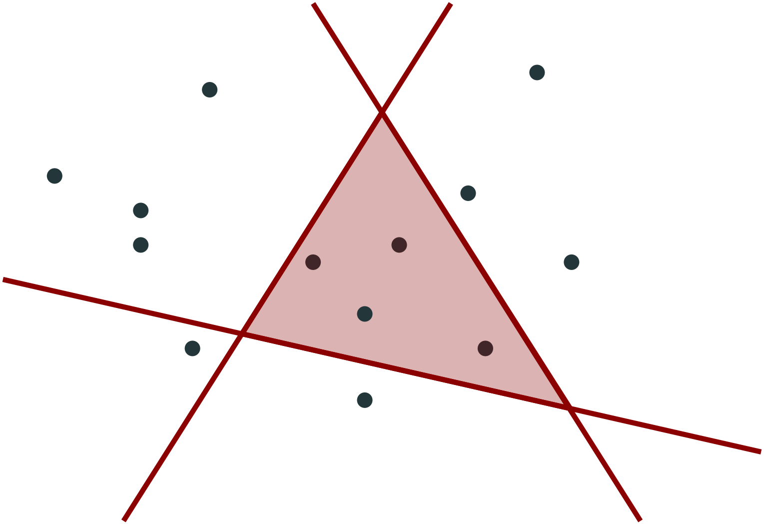
Use higher branching factor of $O(n^{\varepsilon})$ instead of $O(1)$.

Simplex Range Counting - New method



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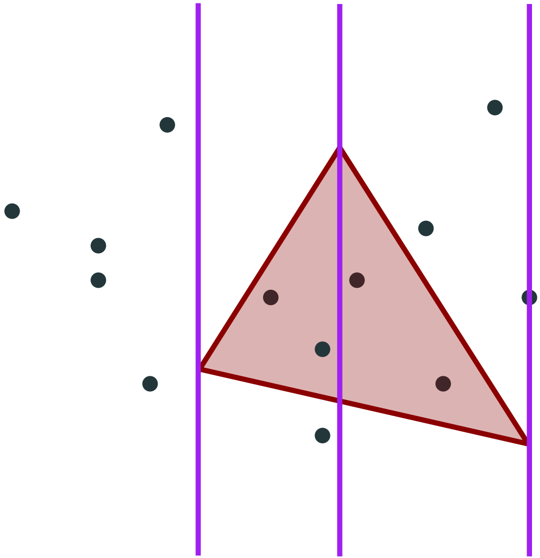

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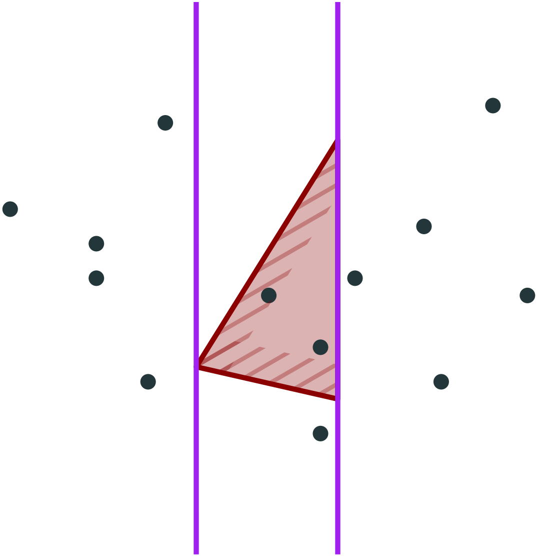

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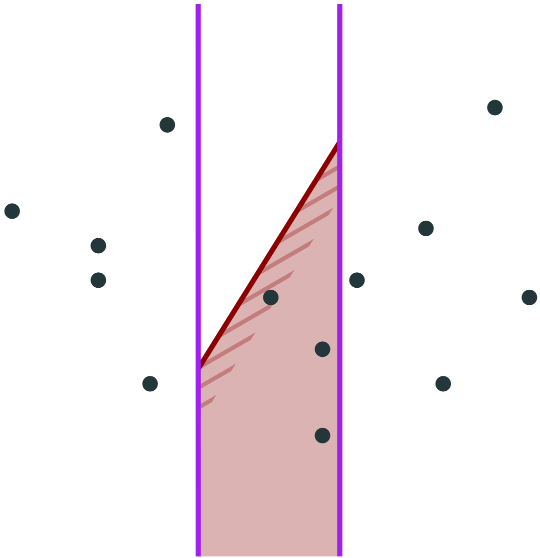

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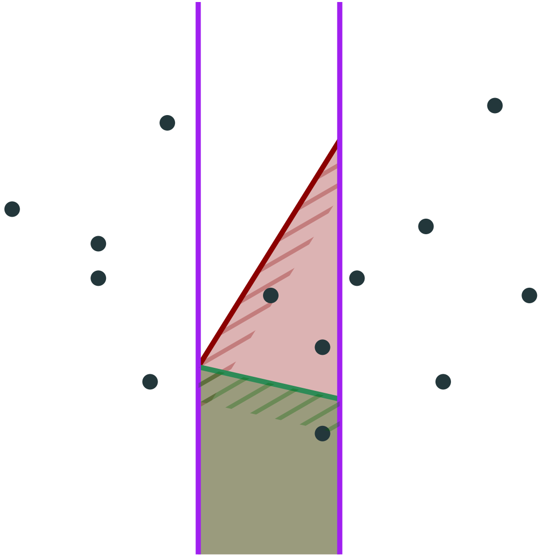
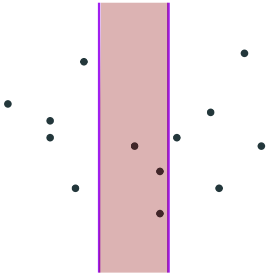


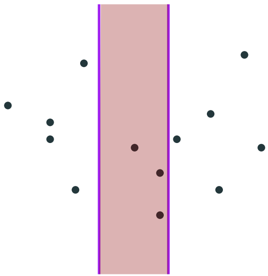
Simplex Range Counting - New method (cont.)

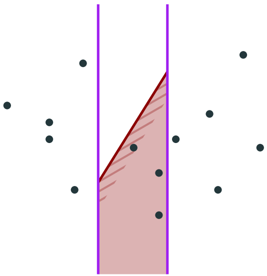
$\mathbb{R}^{d-1}$ simplex counting $\quad O(n^{d-1+\varepsilon})$ space $\quad O(\log n)$ time / query



Simplex Range Counting - New method (cont.)

$\mathbb{R}^{d-1}$ simplex counting	$O(n^{d-1+\varepsilon})$ space	$O(\log n)$ time / query
1-halfspace + $\mathbb{R}^{d-1}$ simplex counting	$O(n^d)$ space	$O(\log^2 n)$ time / query





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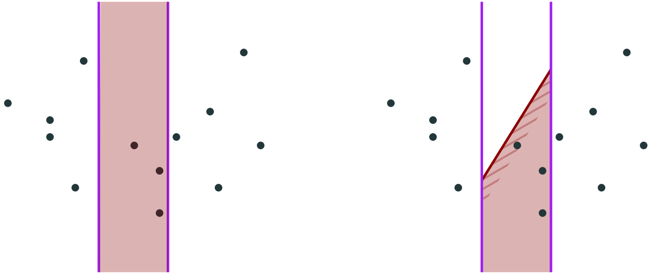
Can reduce runtime to $O(\log n)$ with hierarchical cuttings and n^ε branching factor to ensure outer data structure has $O(1)$ levels.



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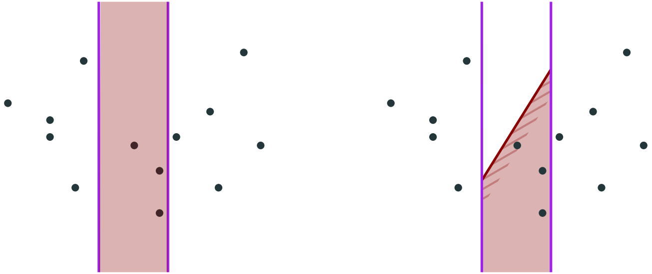
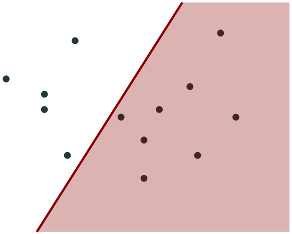


Simplex Range Reporting - New method

1-halfspace reporting $O(n^{\lfloor d/2 \rfloor})$ space $O(\log n + k)$ time / query



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1-halfspace reporting	$O(n^{\lfloor d/2 \rfloor})$ space	$O(\log n + k)$ time / query
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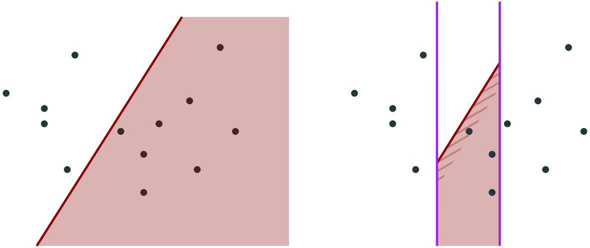

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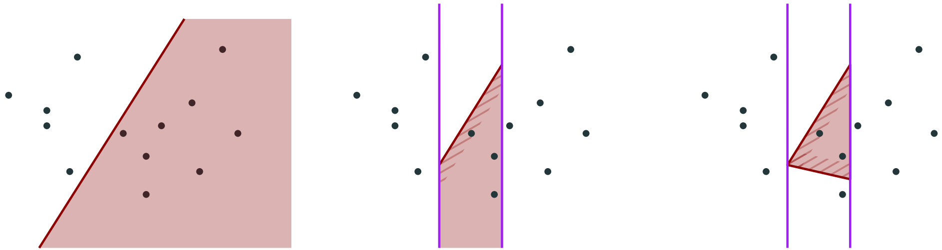
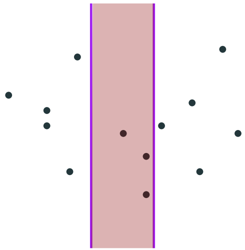
2-halfspace + $\mathbb{R}^{d-1}$ simplex reporting	$O(n^{d})$ space	$O(\log n + k)$ time / query



Simplex Range Semigroup - New method

After some initial point location queries that take $O(\log n)$ time:

$\mathbb{R}^{d-1}$ simplex semigroup $O(n^{d-1+\varepsilon})$ space $O(1)$ time / query



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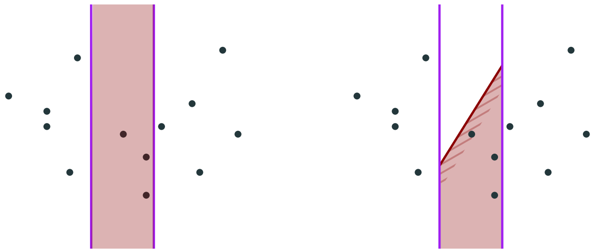

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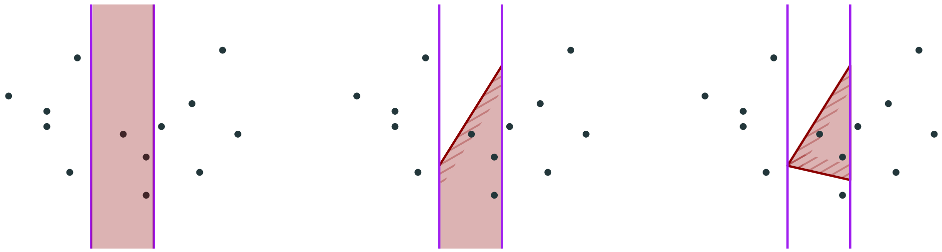

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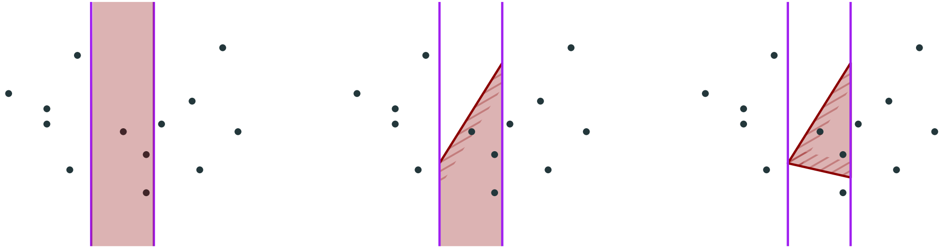
New ideas similar to fractional cascading are needed.



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  - and many more ...

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Open Questions

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Open Questions

- Better trade-offs?

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Open Questions

- Better trade-offs?
- Remove polylog factors for simplex semigroup range searching?

Thanks for listening!