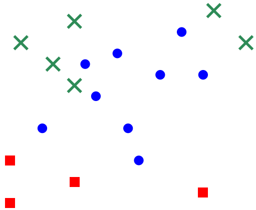
Halving by a Thousand Cuts or Punctures

Weak ε-cuttings and ε-nets for corridors



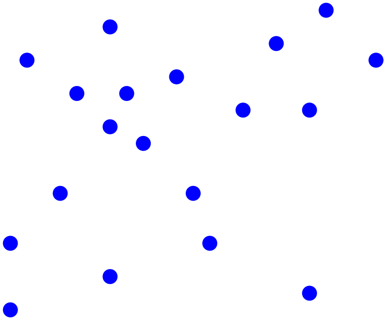
Sariel Har-Peled and **Da Wei Zheng**

February 14, 2023 (NYC Geometry Seminar)

University of Illinois Urbana-Champaign

Presented at SODA 2023

Cutting in Half



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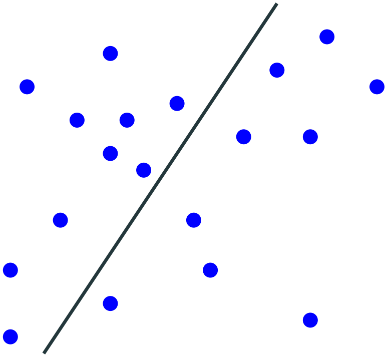

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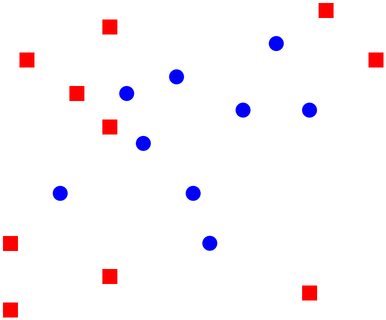

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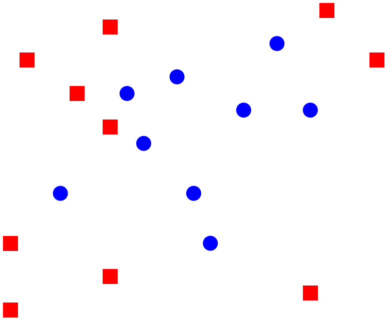

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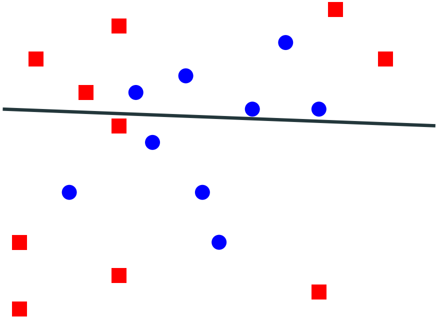

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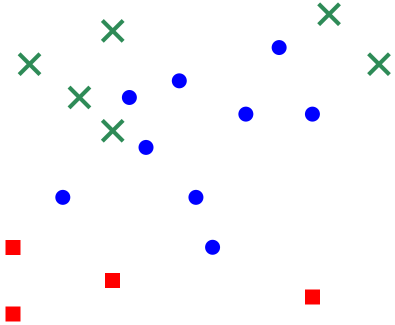

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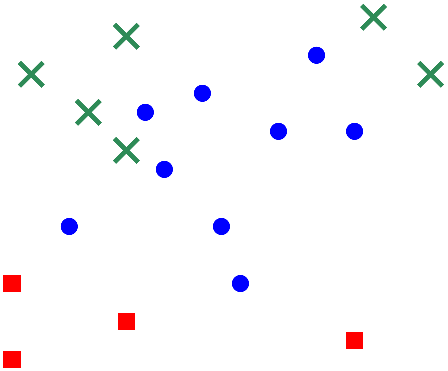

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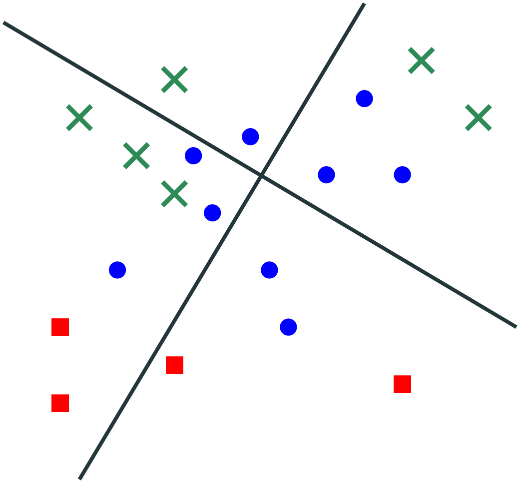

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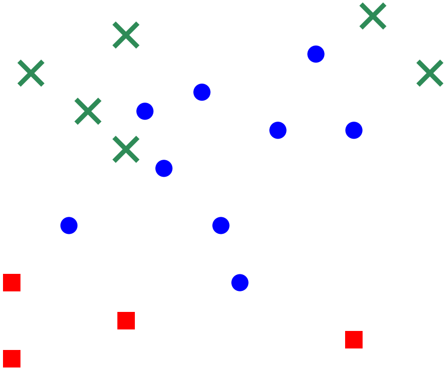

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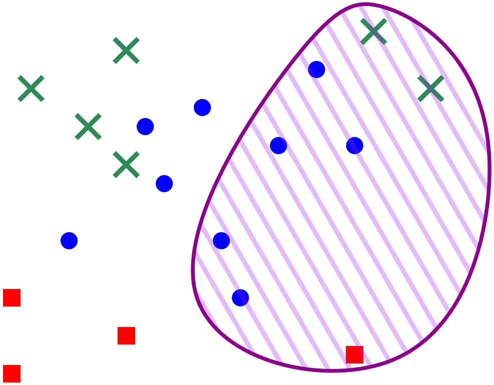

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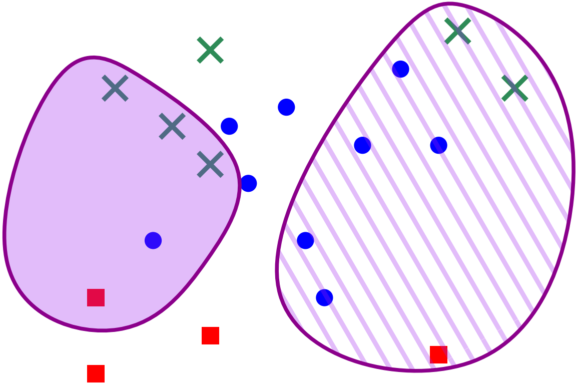

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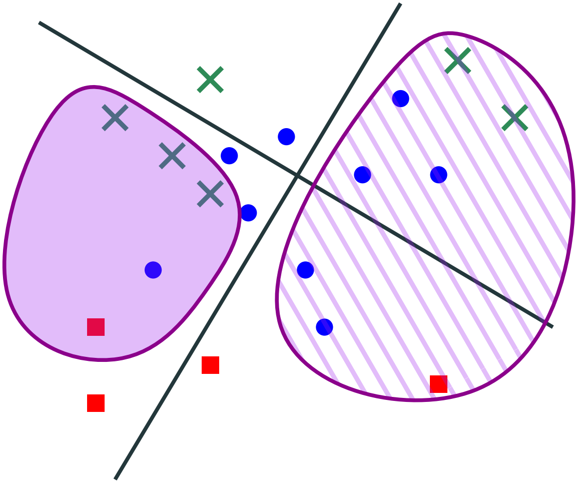

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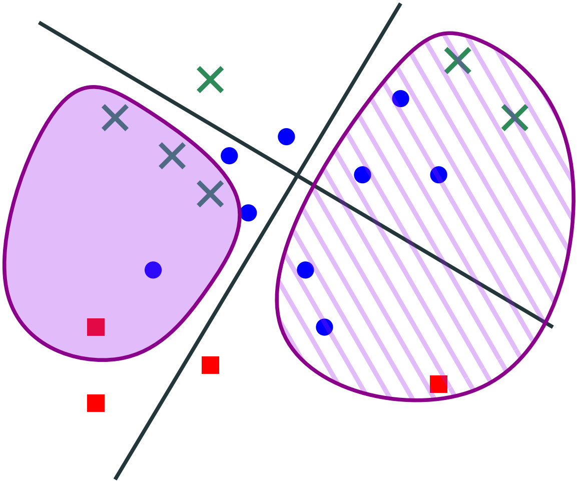
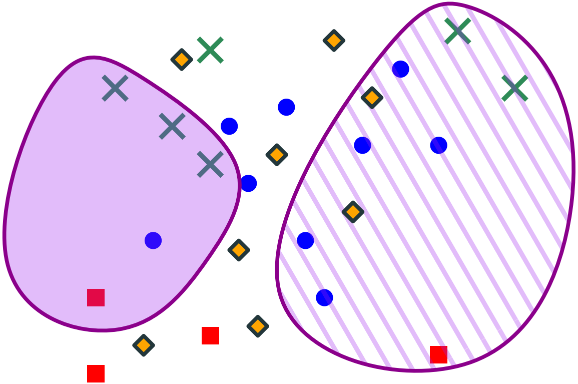


Hitting all the bad convex sets with lines.

The guarding problem



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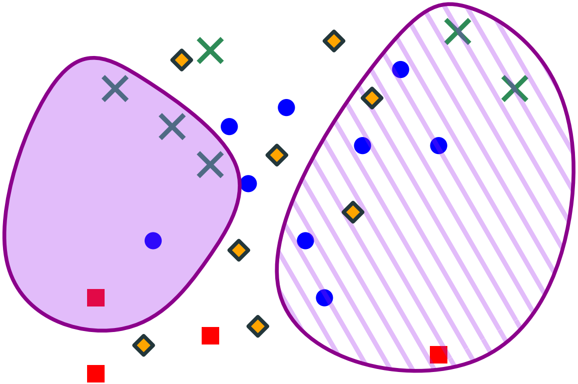
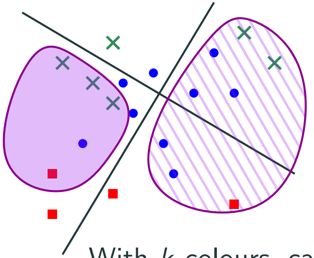
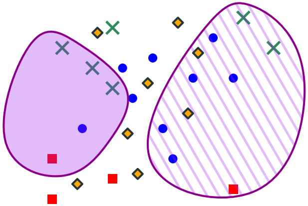


Hitting all the bad convex sets with points.

Known Results

With k colours, can view as k parallel set cover instances.

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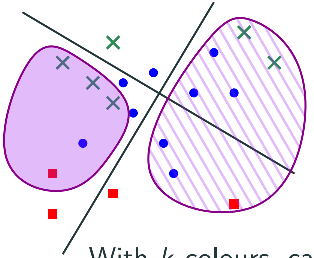

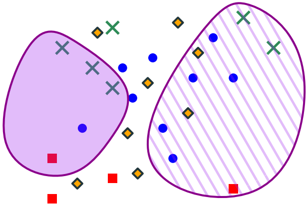

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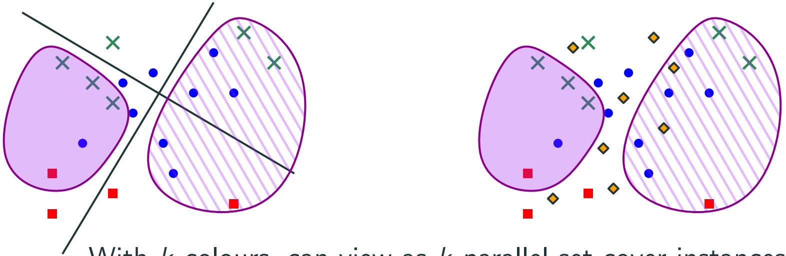

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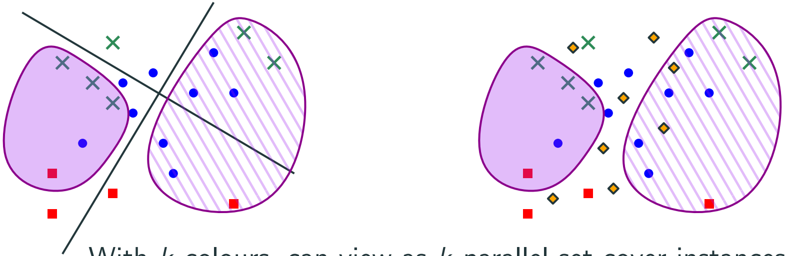

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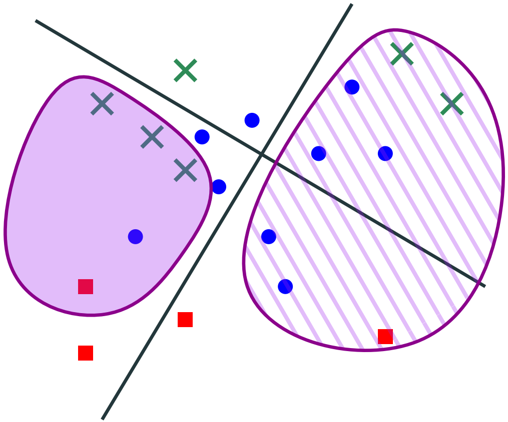
The Linear Program



$$\min_{L} \sum_{\ell \in L} x_\ell$$

$$1 \geq x_\ell \geq 0$$

$$\sum_{\ell \in L \sqcap \sigma} x_\ell \geq 1 \qquad \forall \sigma \in \mathcal{D}_{bad}$$

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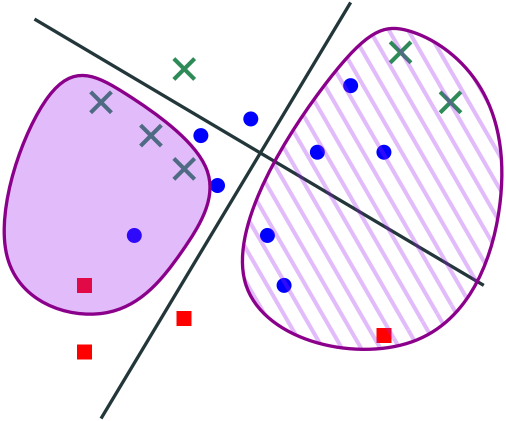

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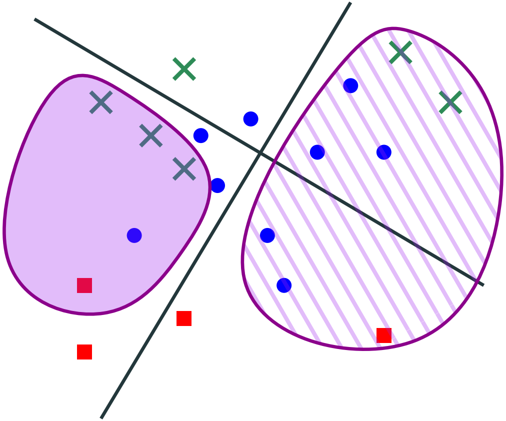

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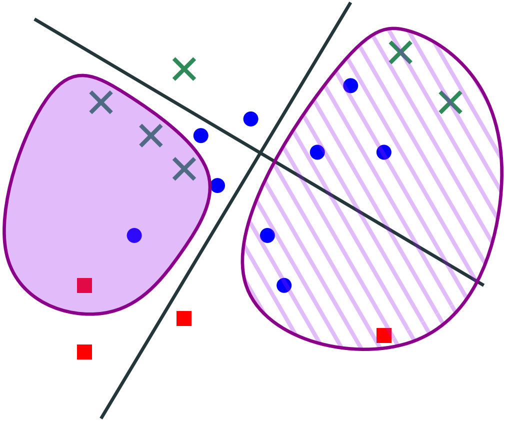

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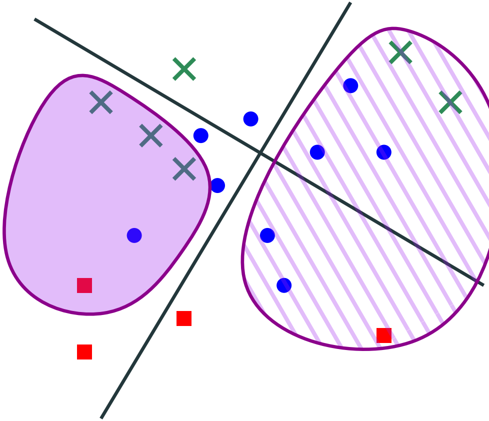

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Rounding the final solution?

ε-nets and ε-cuttings

ε-nets and random sampling

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Definition

A subset $S \subseteq X$ is an ε-net for X (against $\mathcal{H}$), if for every range $h \in \mathcal{H}$ where $|X \cap h| \geq \varepsilon|X|$, there is some $s \in S$ that lies in h (i.e. $s \cap h \neq \emptyset$).

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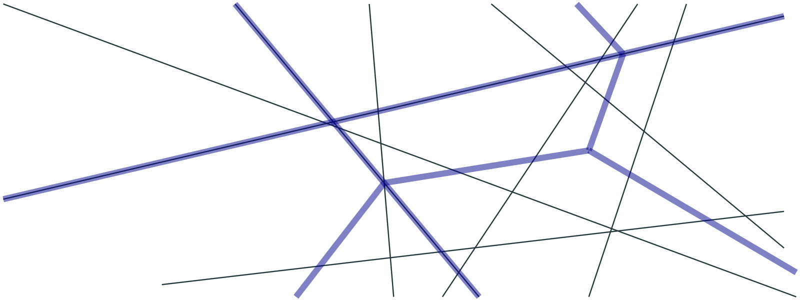
Theorem (Haussler and Welzl '82)

Let $(X, \mathcal{H})$ be a range space of VC dimension d, and suppose that $0 < \varepsilon \leq 1$ and $0 < \delta < 1$. Then a random sample of size $\Omega(\varepsilon^{-1}(\log \delta^{-1} + d \log \varepsilon^{-1}))$ is a ε-net for X with probability at least $1 - \delta$.

Definition

Given a set of n lines $\mathcal{L}$, and $0 < \varepsilon < 1$, an ε-cutting of $\mathcal{L}$ is a partition of $\mathbb{R}^d$ into connected regions where every region intersects at most εn lines of $\mathcal{L}$.



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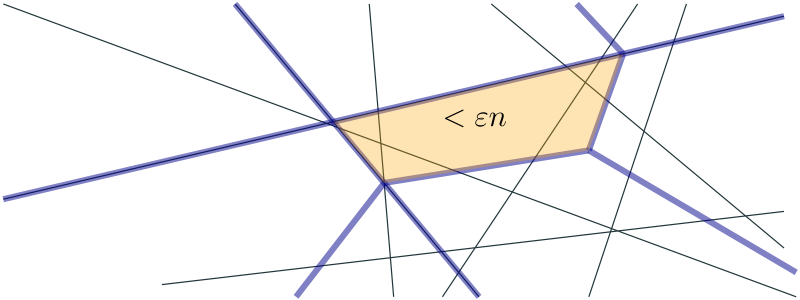

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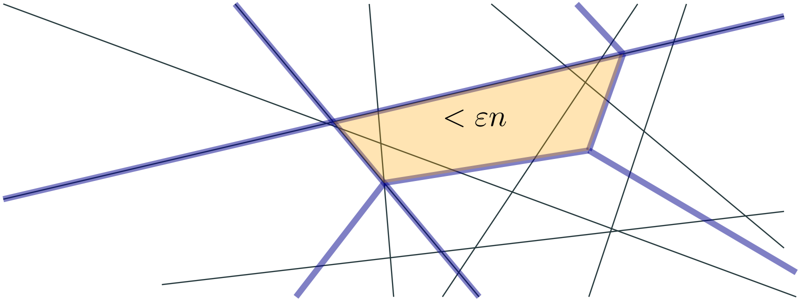
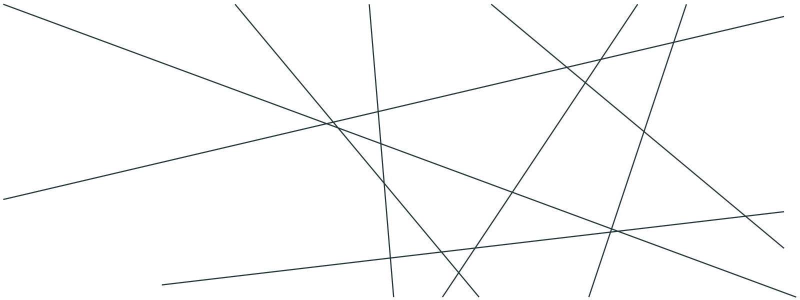
Theorem (Matoušek '91, Chazelle '93)
For any set of lines $\mathcal{L}$ in $\mathbb{R}^d$, there exist cuttings of size $O(1/\varepsilon^d)$.



Weak ε-cuttings



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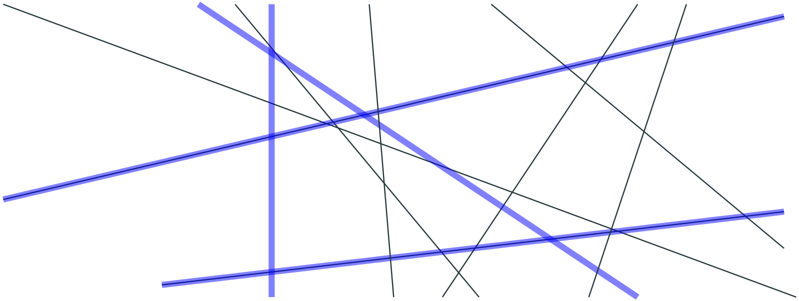

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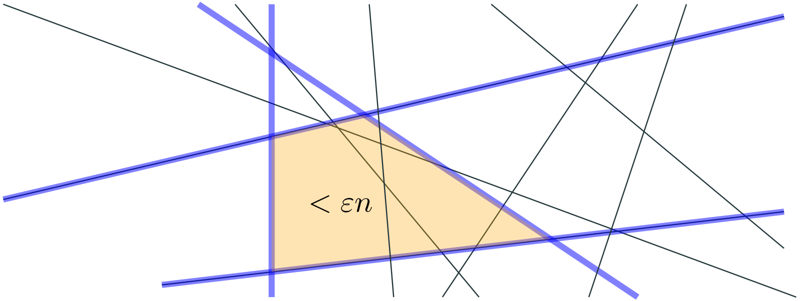

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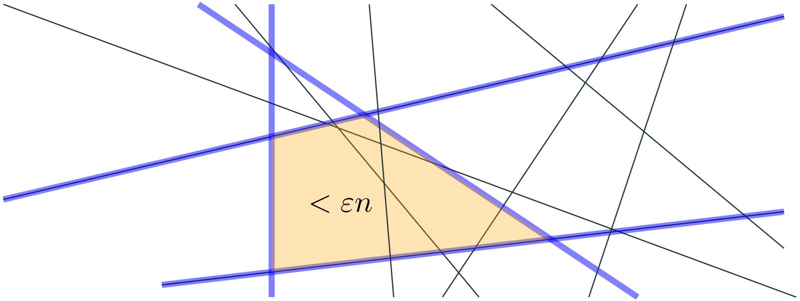
For a set P of n lines in $\mathbb{R}^2$, a set $\mathcal{L}$ of lines is a weak ε-cutting if for every convex region D where $|D \sqcap P| \geq \varepsilon n$ intersects at least one line of $\mathcal{L}$.



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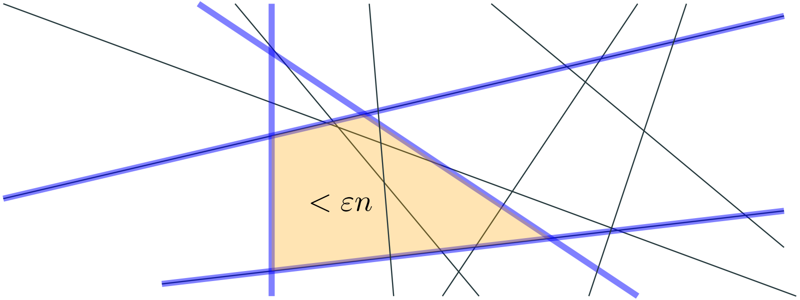

Weak ε-nets for convex sets

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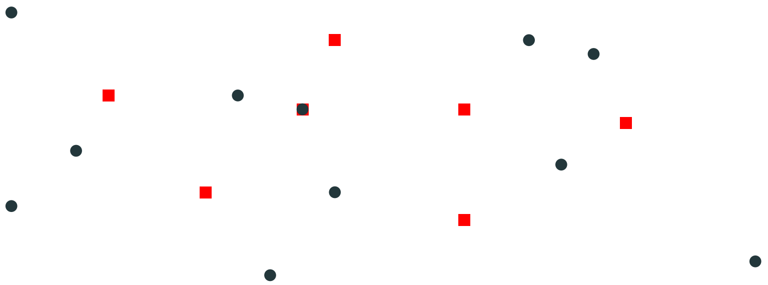

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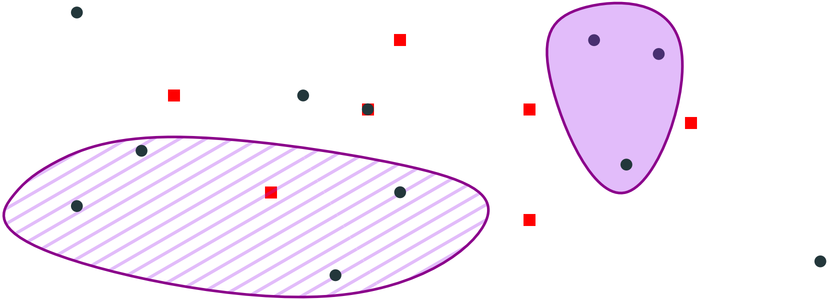
For a set P of n points in $\mathbb{R}^2$, a set $S \subset \mathbb{R}^2$ is a weak ε-net for convex sets if for every convex region D where $|D \cap P| \geq \varepsilon n$ has nonempty intersection with S.



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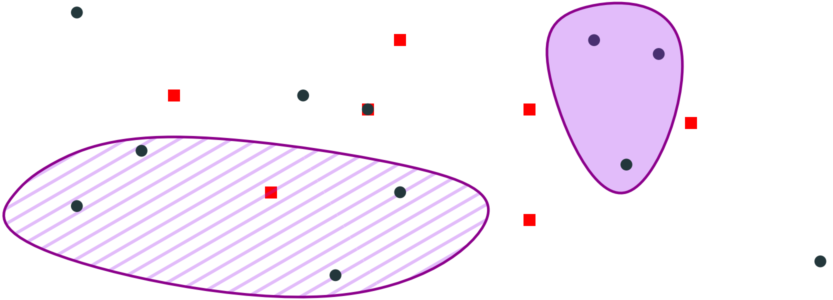
Theorem (Bárány et al '90, Alon et al '92, Rubin '18)
For any n points in $\mathbb{R}^2$ and $\varepsilon, \alpha > 0$, there exists a weak ε-net of size $O\left(1/\varepsilon^{3/2+\alpha}\right)$.



Weak ε-cuttings (Again)

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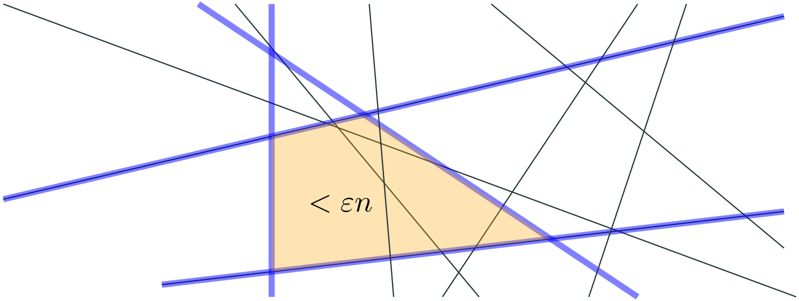

An observation about weak ε-cuttings.

Corridors

Definition

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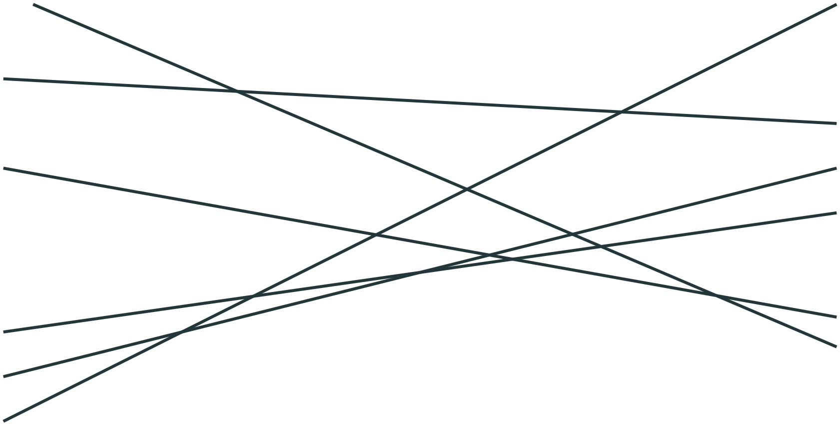

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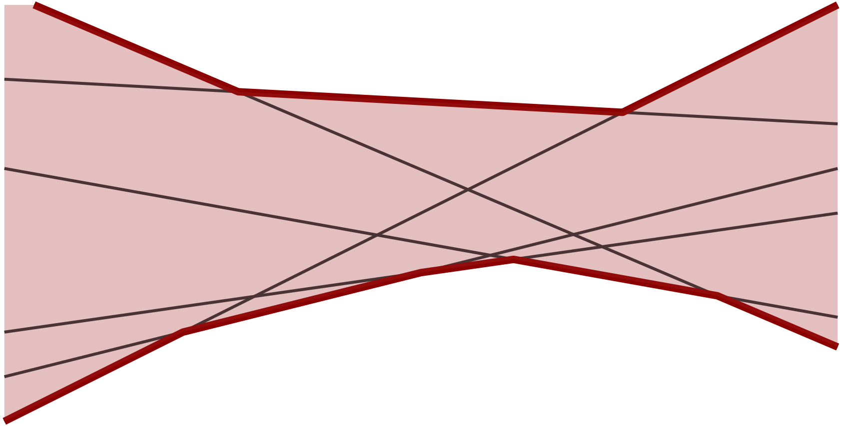
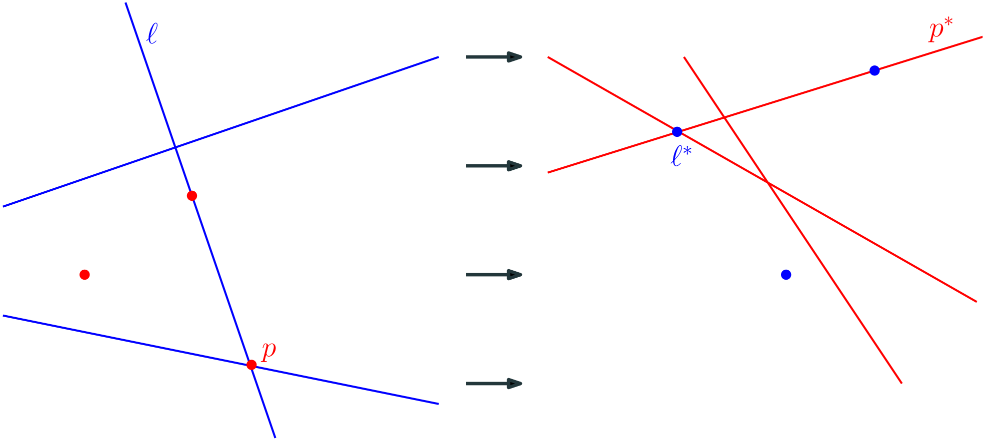


Projective Duality for Convex Hulls

Projective Duality - Transform that takes points to lines and lines to points that preserves incidences and above-below relationships.



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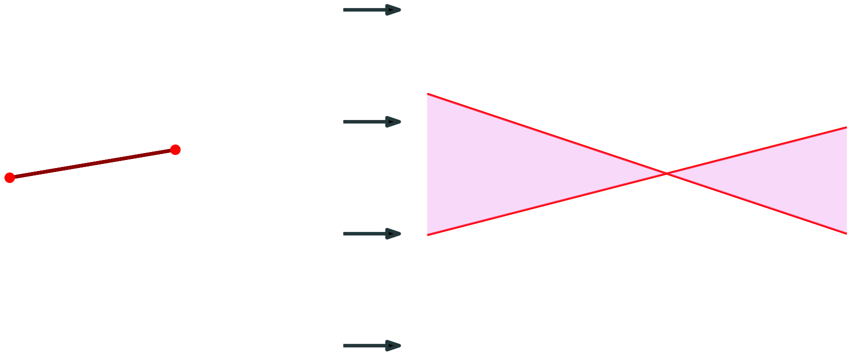

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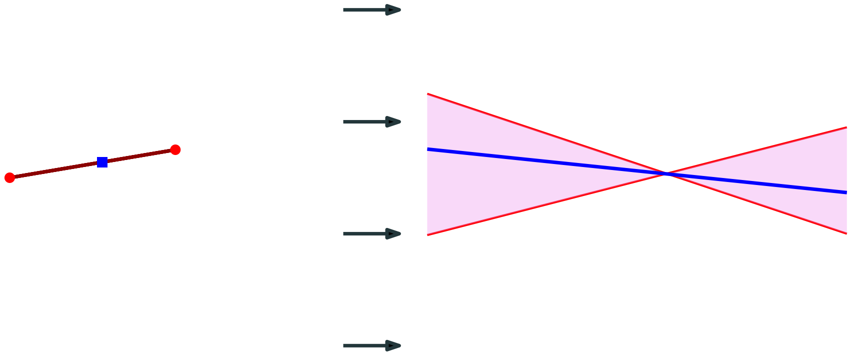

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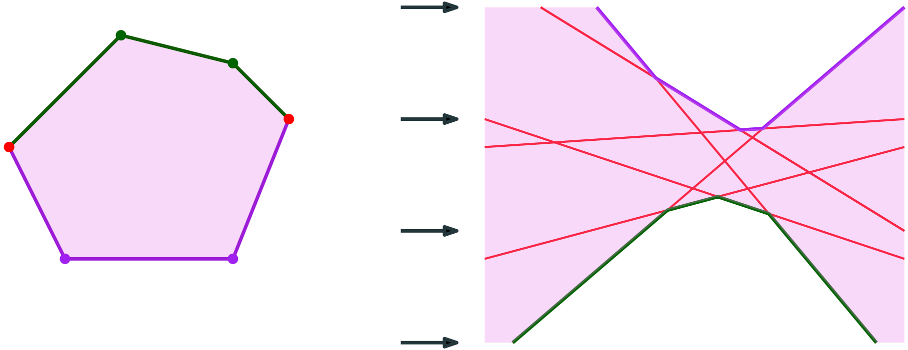

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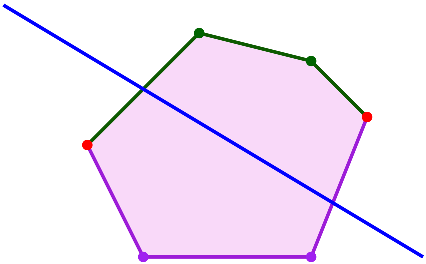

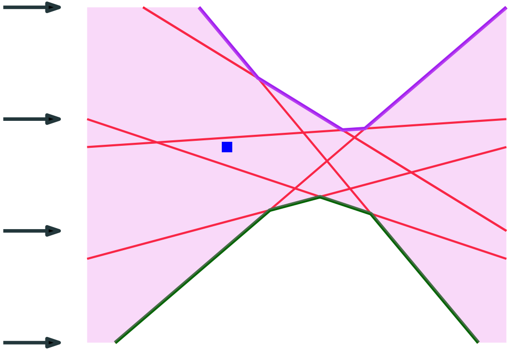

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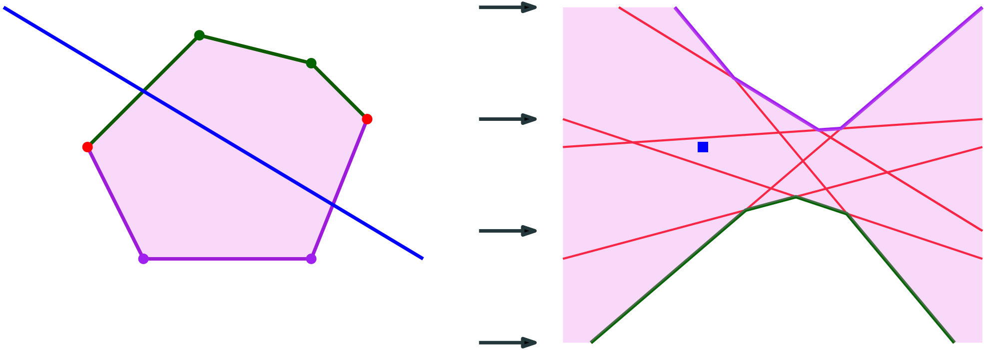
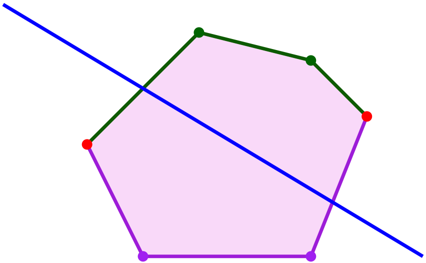
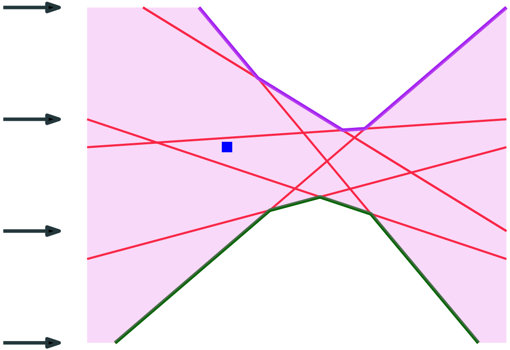


Line intersecting convex hull $\rightarrow$ Point in corridor





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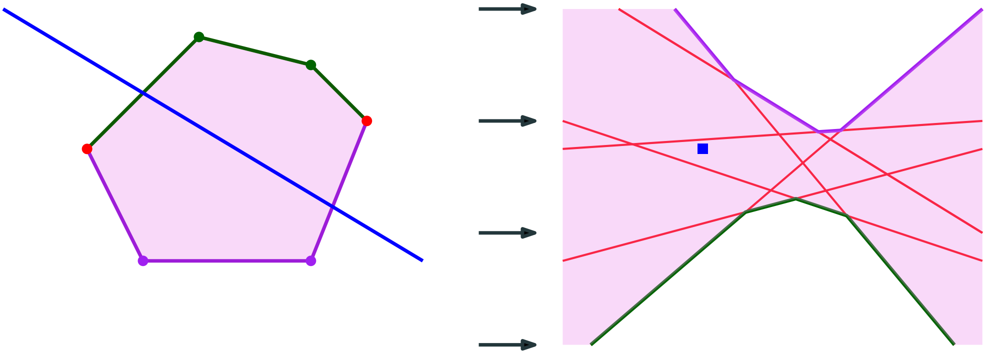

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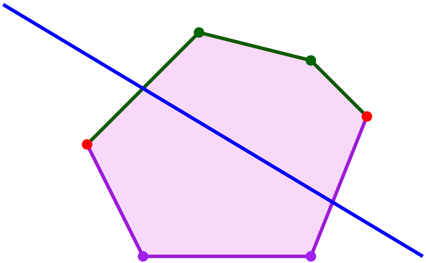

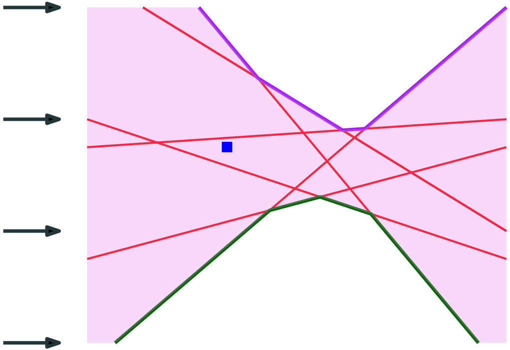

Theorem
For any n lines in $\mathbb{R}^2$ *and* $\varepsilon > 0$*, exists a weak* ε*-cutting of size* $\widetilde{O}\left(1/\varepsilon^{3/2}\right)$.

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For any n points in $\mathbb{R}^2$ *and* $\varepsilon > 0$*, exists a weak* ε*-net for corridors of size* $\widetilde{O}\left(1/\varepsilon^{3/2}\right)$.

Warmup - Weak ε-cuttings of size $O(1/\varepsilon^2)$

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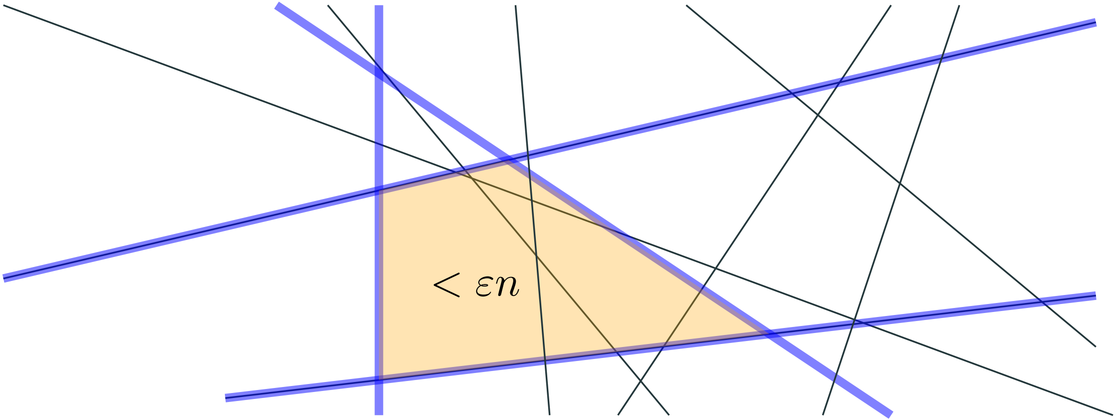

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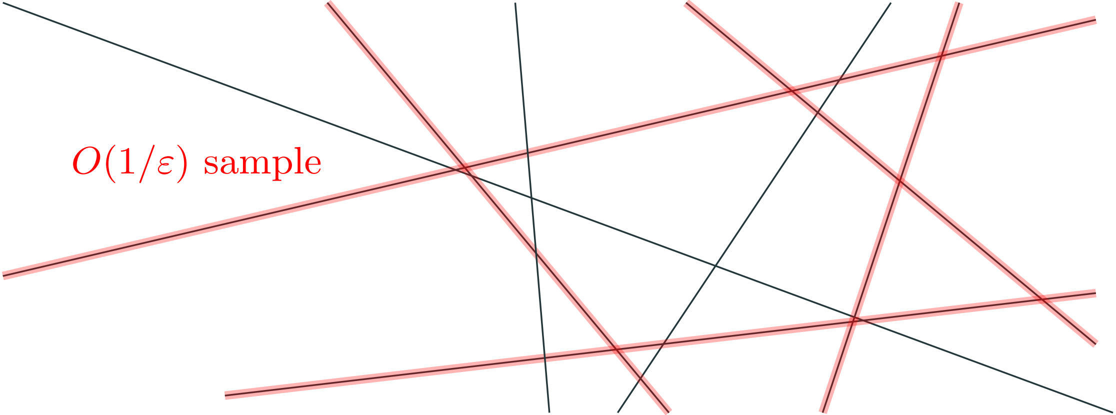

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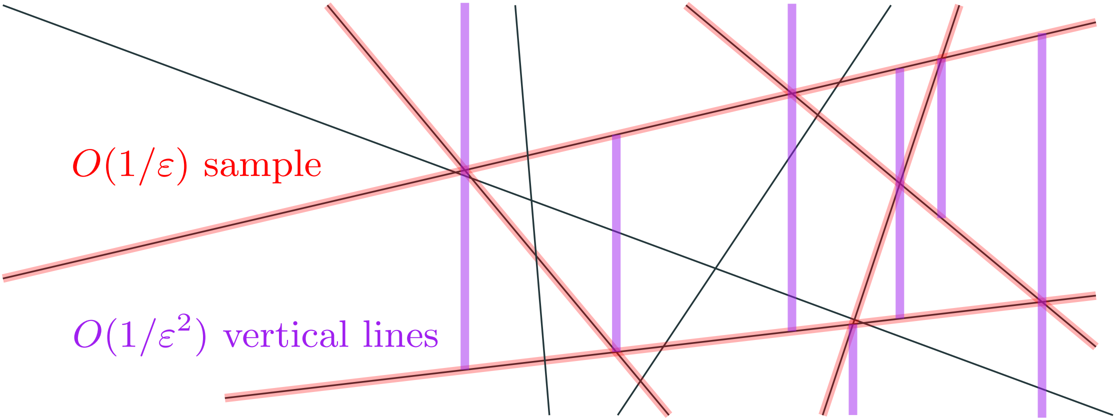

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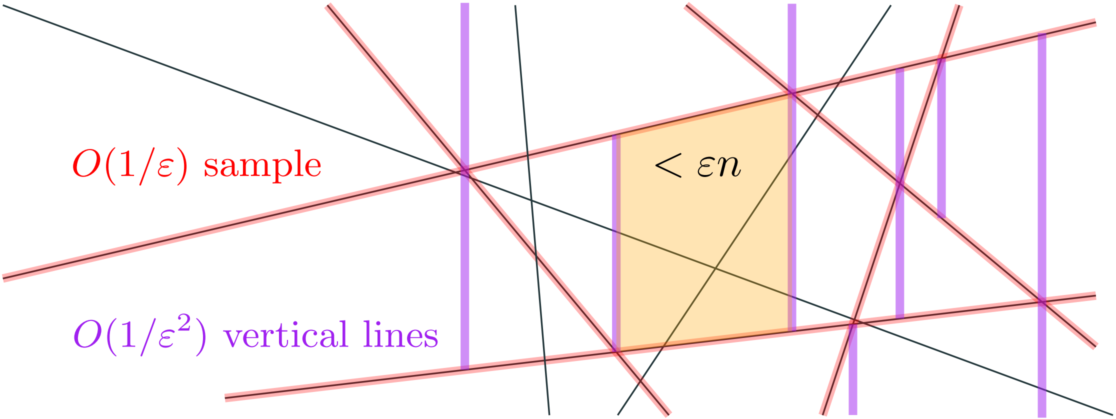

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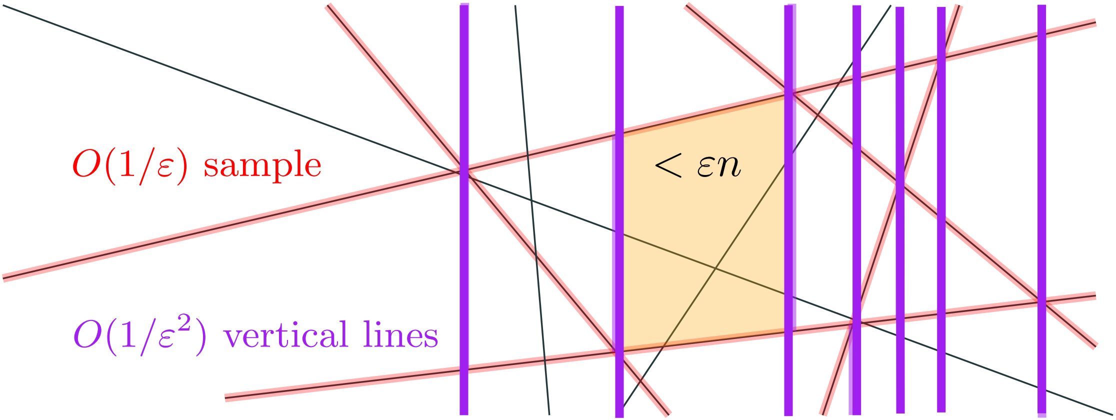

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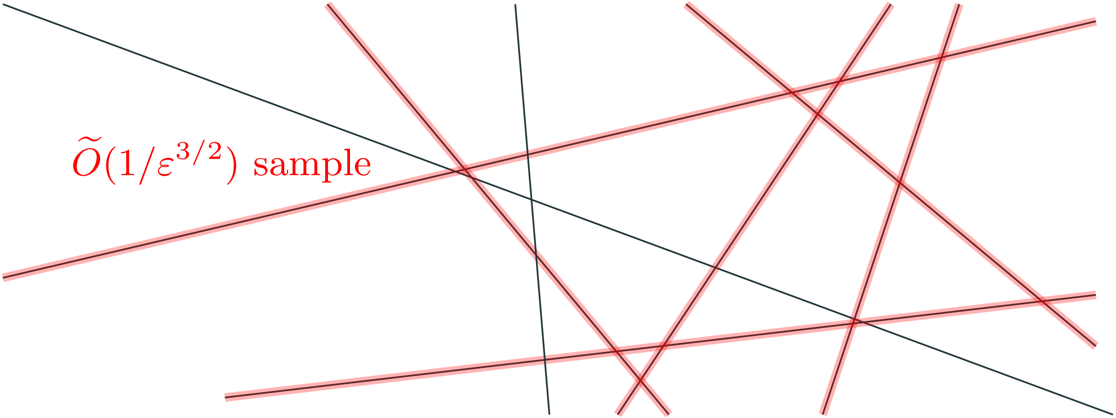

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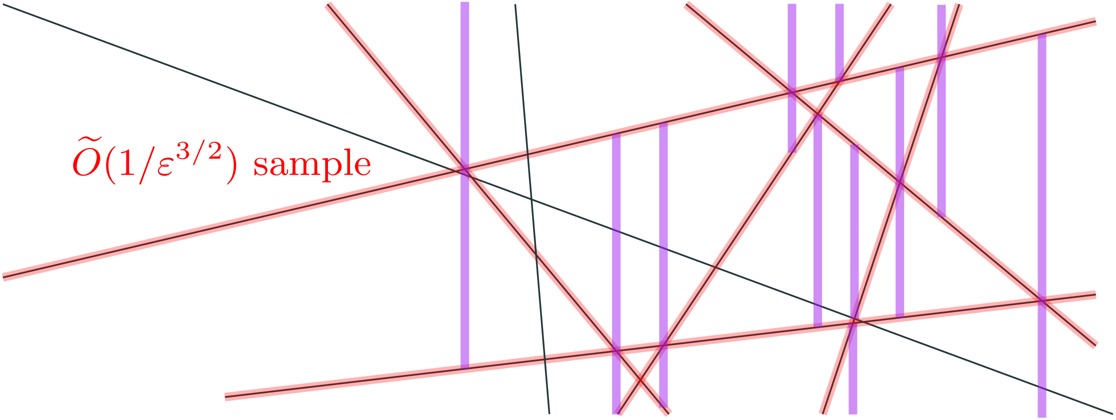

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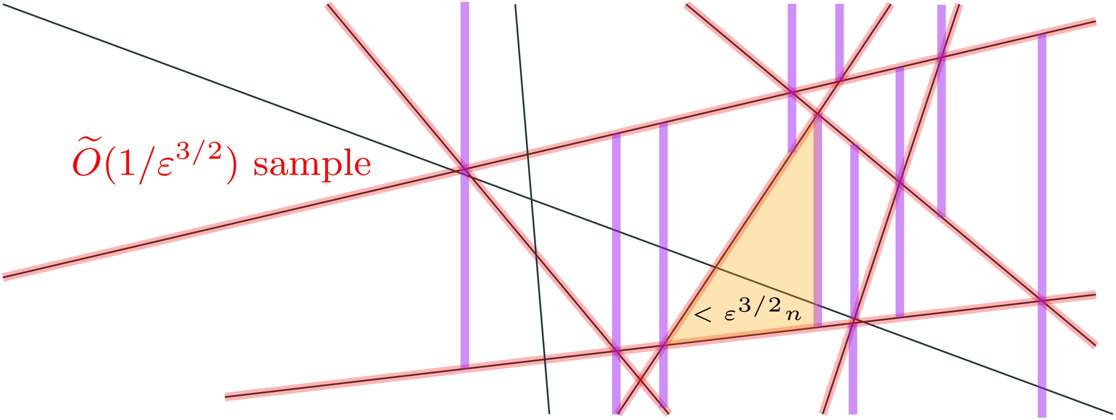

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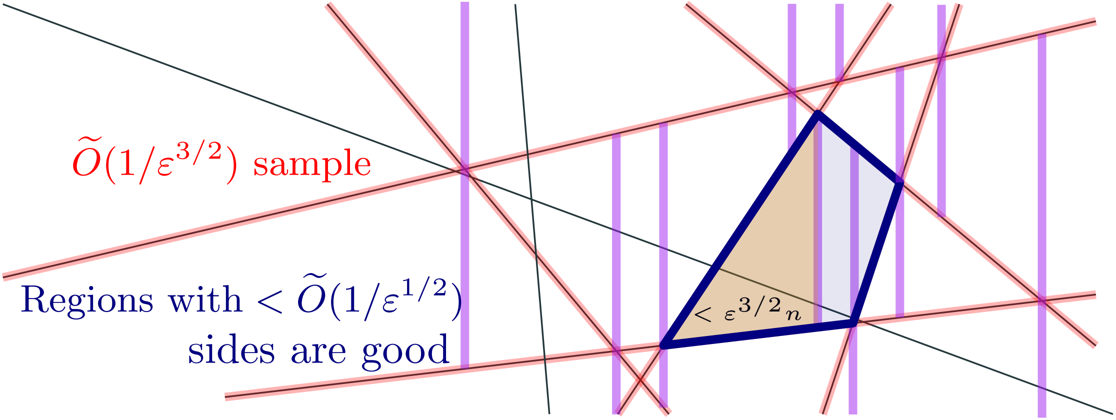

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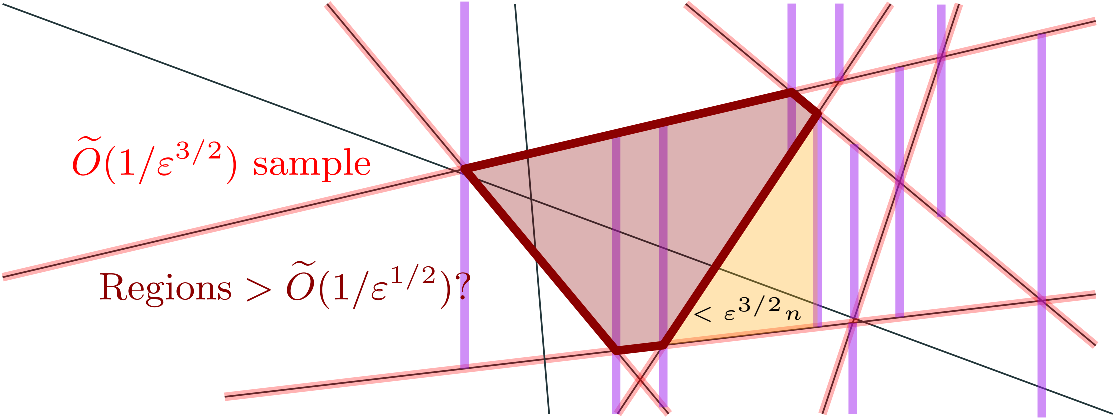

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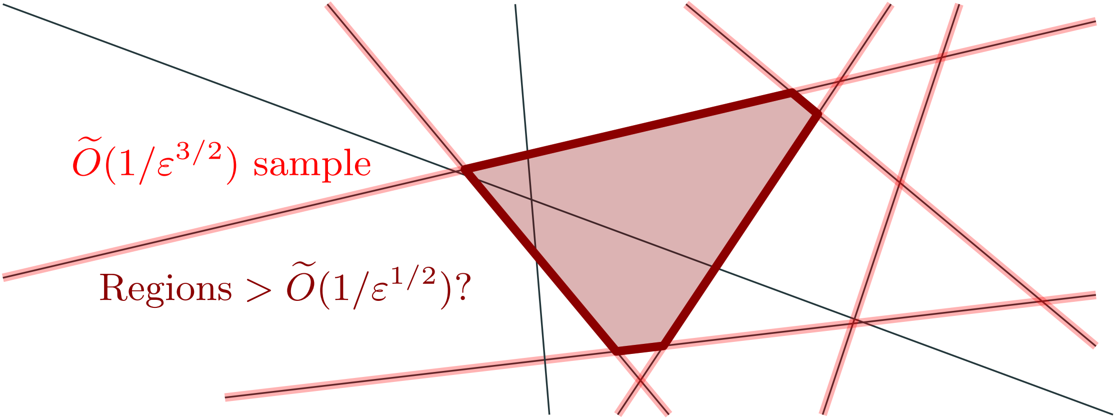

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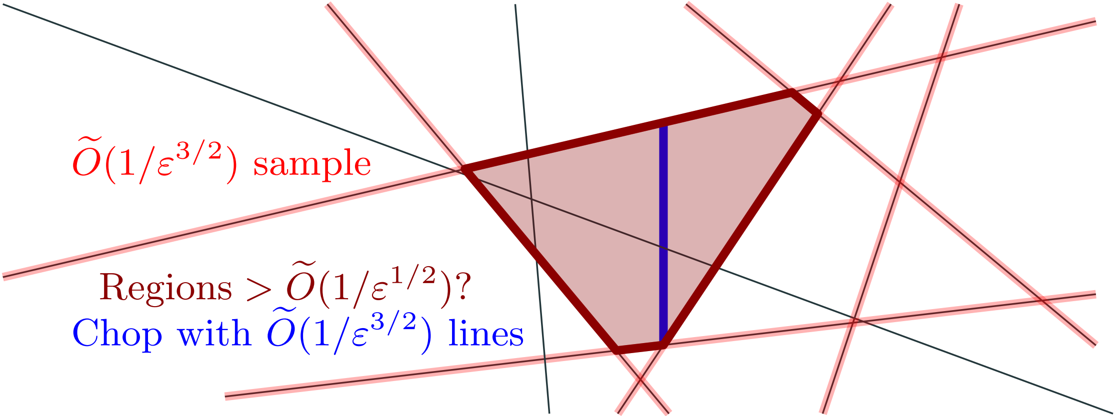

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Cut each big faces into parts of size $\widetilde{O}(r^{1/2})$, then we need to make $\widetilde{O}(r^{3/2})$ cuts.

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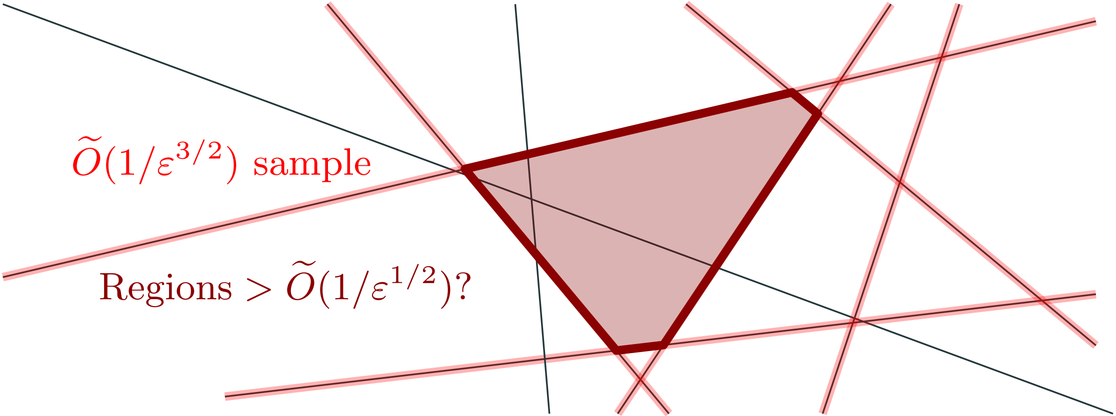

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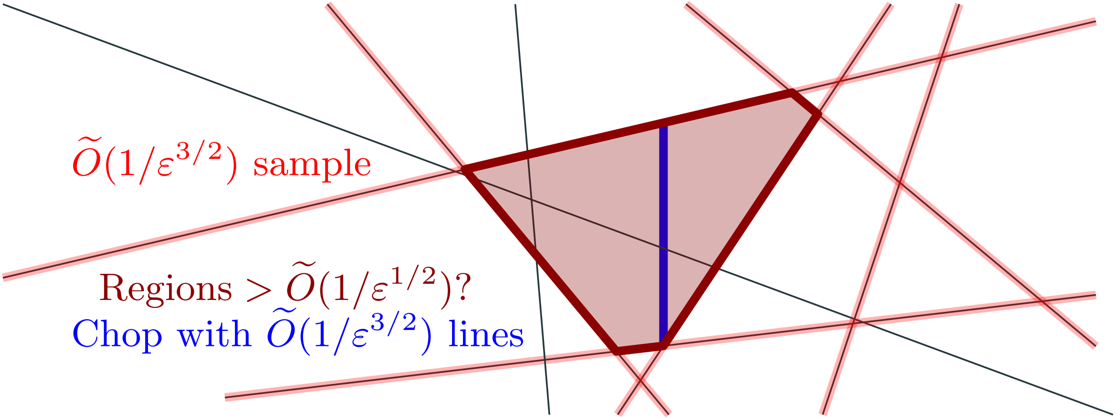

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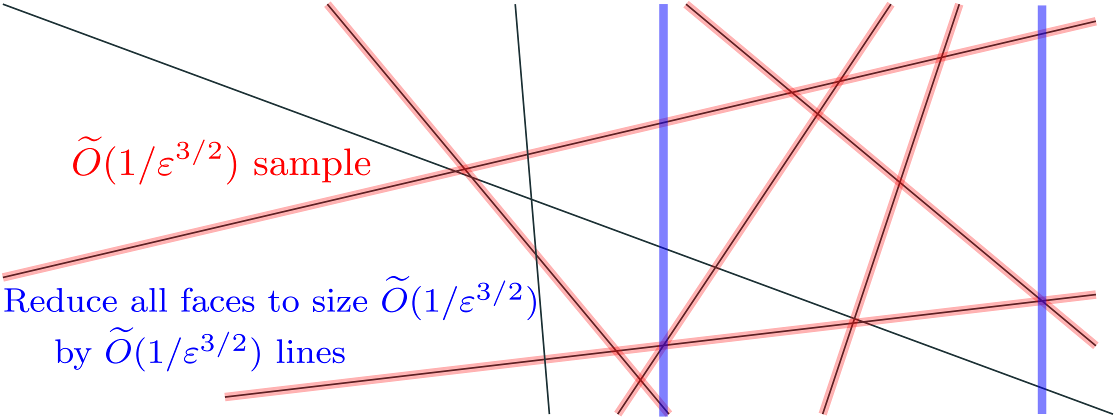
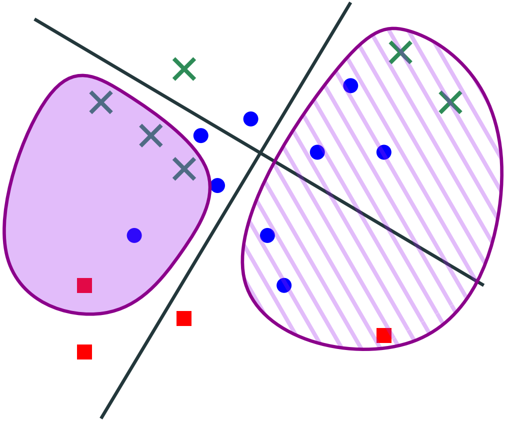


Back to solving the halving problem

The Linear Program



$$\min_{L} \sum_{\ell \in L} x_\ell$$

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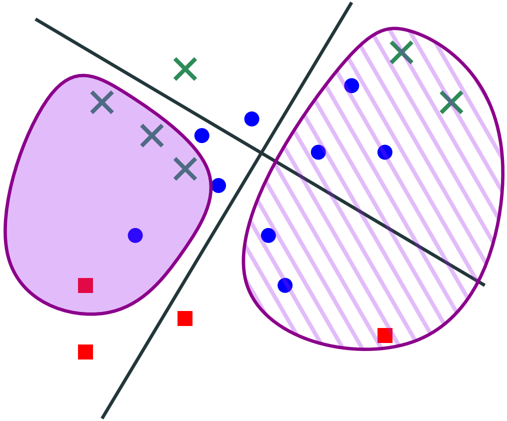

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Issues:

Can there be many lines in L?

Size of $\mathcal{D}_{bad}$?

Separation oracle?

Rounding the final solution?

Rounding Problem

Suppose we had a (possibly fractional) solution L of size t to the problem.

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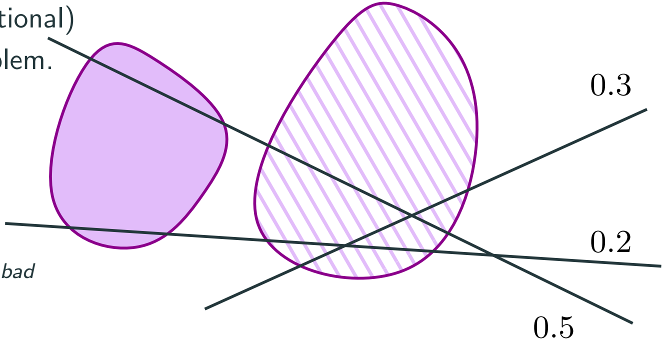

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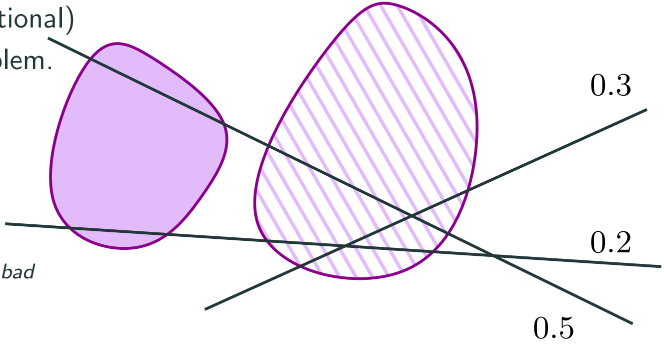

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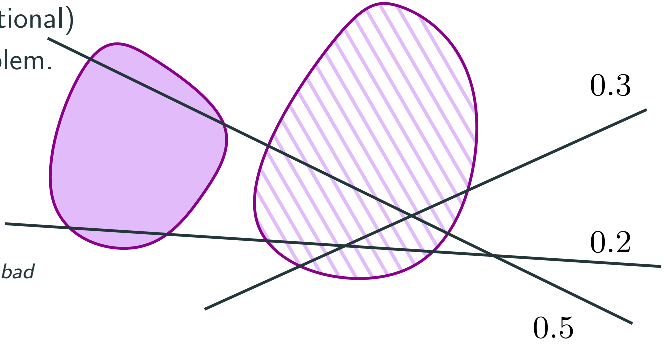

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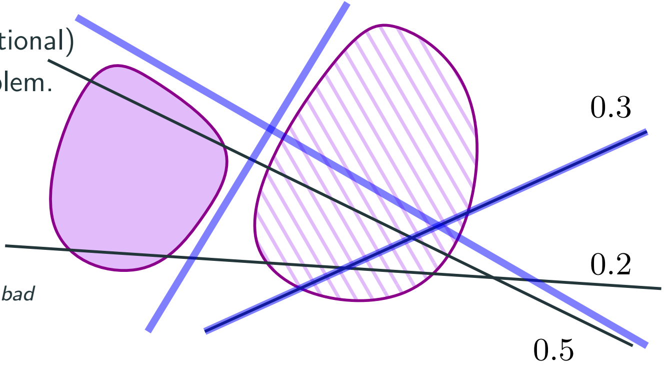

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4. We check each face of the cutting (which must have weight < 1 of the lines of $\mathbf{x}$) against the original point set. One of the following happens:
   (a) It is a halving set for each coloured point set. (Done!)
   (b) We find a convex face of the arrangment that has too many points of some colour, and it also has weight < 1 of the lines of $\mathbf{x}$. (Separation!)

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- We can round the LP (and solve the halving problem) to get a solution of size $\widetilde{O}(OPT^{3/2})$.

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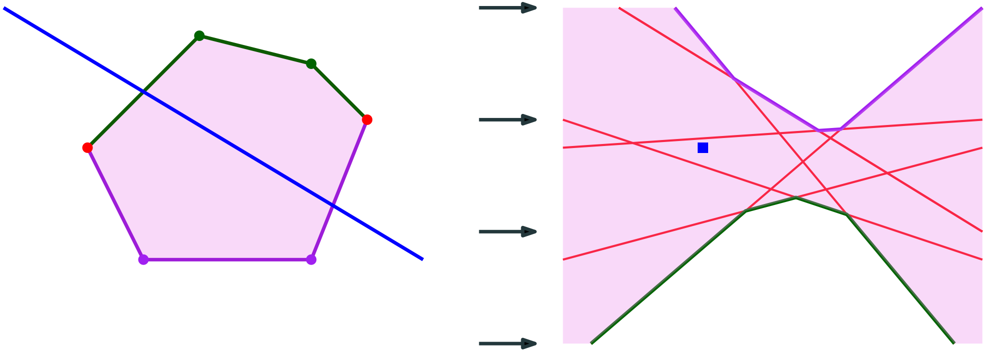
- Improve the exponent of $3/2$.

Thanks for listening!



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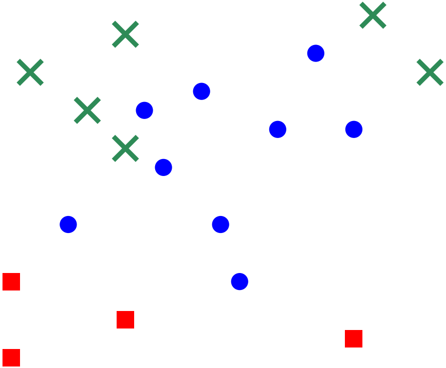

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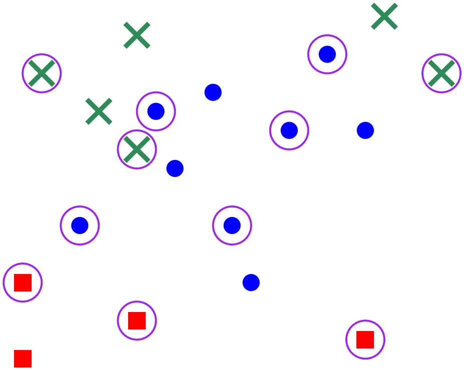

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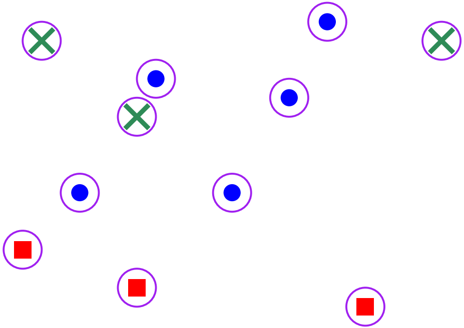

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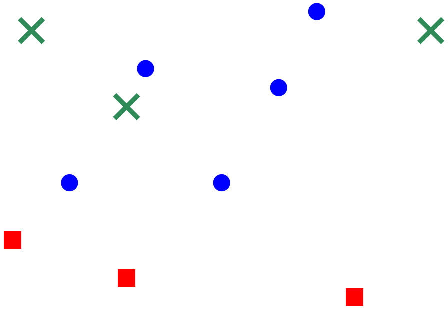

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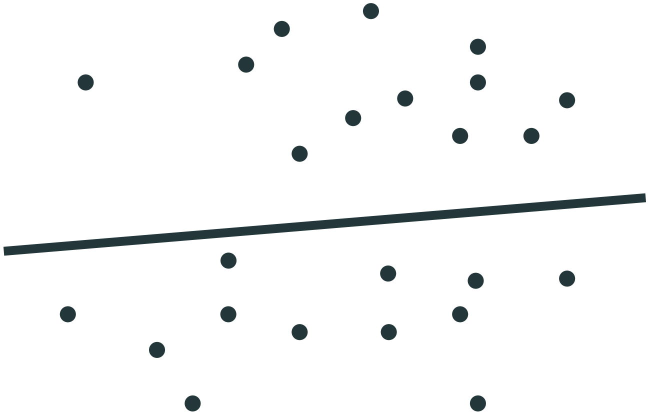

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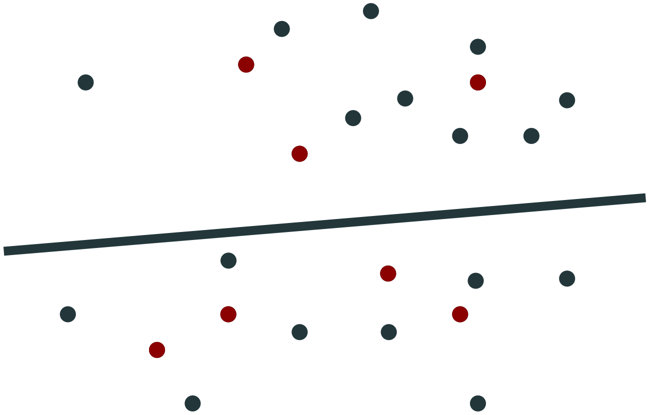

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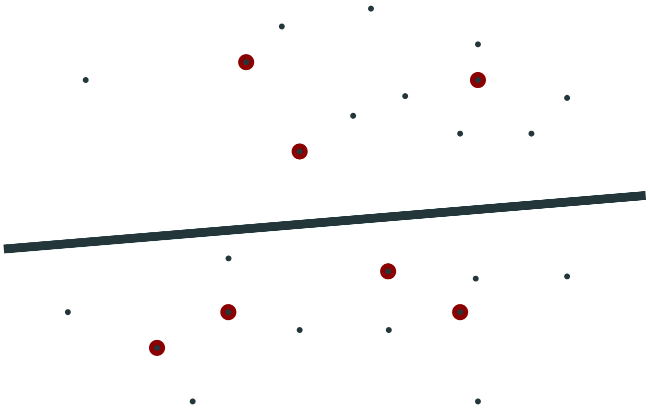

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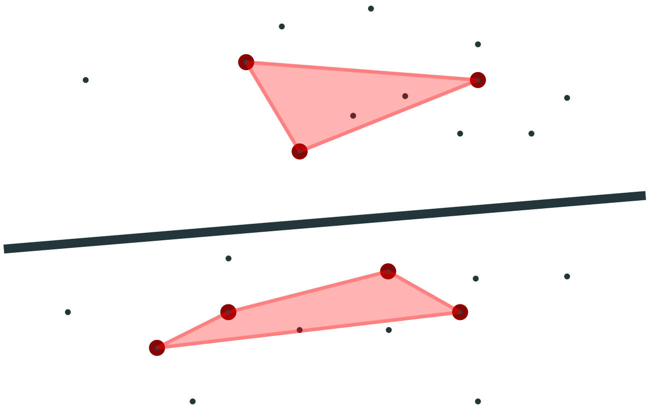

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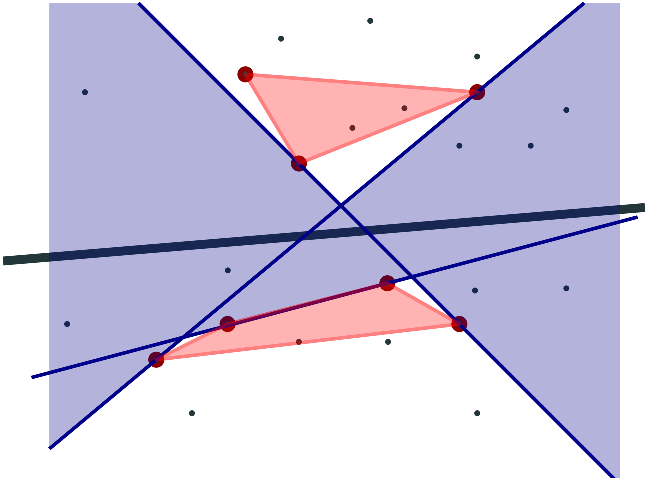

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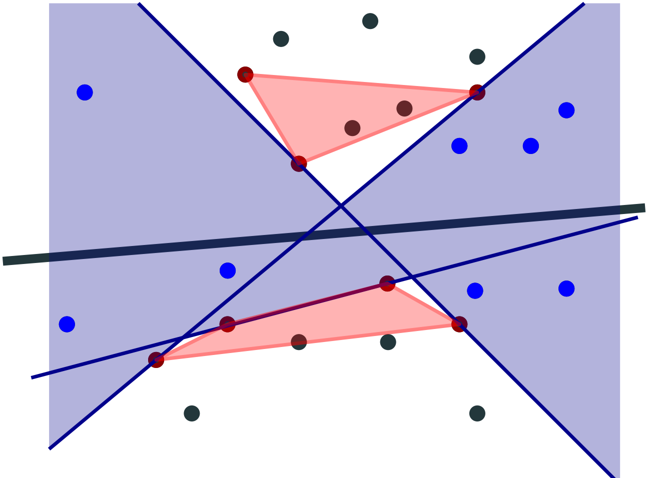

A brief aside about projective duality

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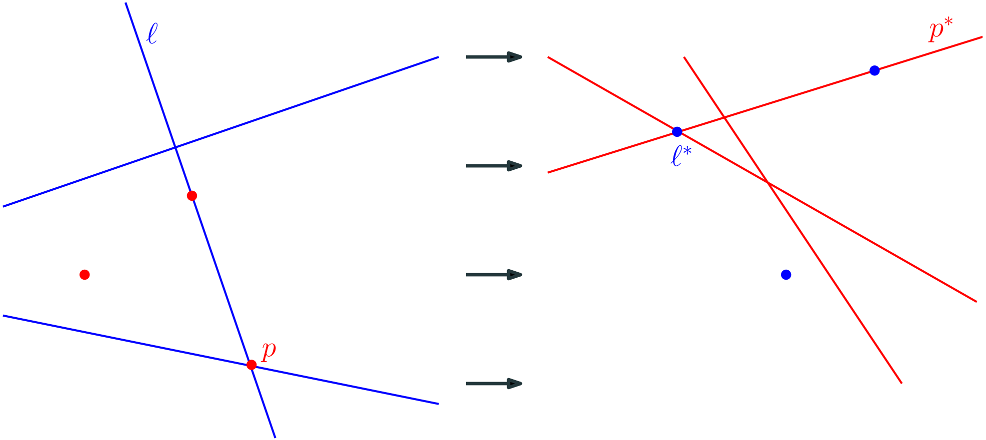
Projective Duality - Transform that takes points to lines and lines to points that preserves incidences and above-below relationships.



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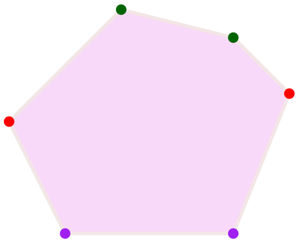

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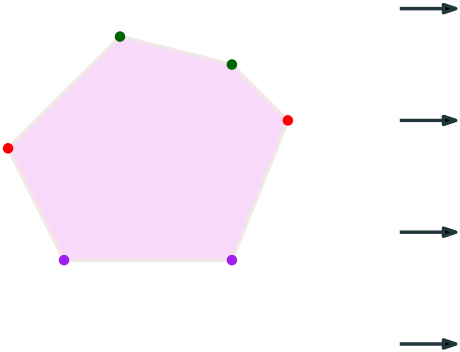
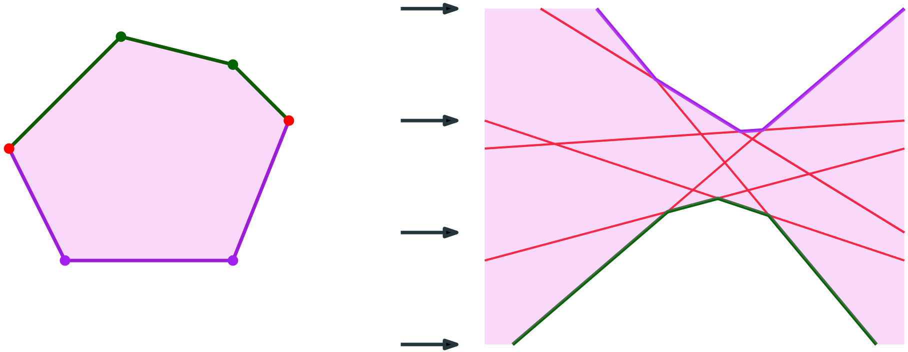


Projective Duality - Transform that takes points to lines and lines to points that preserves incidences and above-below relationships.



Reducing the number of lines

Idea: Sample points of each colour and only consider lines passing through pairs of sampled points.